JEPPIAAR INSTITUTE OF TECHNOLOGY

"Self-Belief | Self Discipline | Self Respect"

DEPARTMENT

OF

ELECTRONICS & COMMUNICATION ENGINEERING

LECTURE NOTES

EC8352/ SIGNALS AND SYSTEMS

(Regulation 2017)

Year/Semester: II/III ECE

2021 – 2022

Prepared by

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Assistant Professor/ECE

EC8352 **SIGNALS AND SYSTEMS** **L T P C**
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OBJECTIVES:

- To understand the basic properties of signal & systems
- To know the methods of characterization of LTI systems in time domain
- To analyze continuous time signals and system in the Fourier and Laplace domain
- To analyze discrete time signals and system in the Fourier and Z transform domain

UNIT I - CLASSIFICATION OF SIGNALS AND SYSTEMS 12
Standard signals- Step, Ramp, Pulse, Impulse, Real and complex exponentials and Sinusoids_ Classification of signals – Continuous time (CT) and Discrete Time (DT) signals, Periodic & Aperiodic signals, Deterministic & Random signals, Energy & Power signals - Classification of systems- CT systems and DT systems- – Linear & Nonlinear, Time-variant & Time-invariant, Causal & Non-causal, Stable & Unstable.

UNIT II - ANALYSIS OF CONTINUOUS TIME SIGNALS 12
Fourier series for periodic signals - Fourier Transform – properties- Laplace Transforms and properties

UNIT III - LINEAR TIME INVARIANT CONTINUOUS TIME SYSTEMS 12
Impulse response - convolution integrals- Differential Equation- Fourier and Laplace transforms in Analysis of CT systems - Systems connected in series / parallel.

UNIT IV ANALYSIS OF DISCRETE TIME SIGNALS 12
Baseband signal Sampling – Fourier Transform of discrete time signals (DTFT) – Properties of DTFT - Z Transform & Properties

UNIT V LINEAR TIME INVARIANT-DISCRETE TIME SYSTEMS 12
Impulse response – Difference equations-Convolution sum- Discrete Fourier Transform and Z Transform Analysis of Recursive & Non-Recursive systems-DT systems connected in series and parallel.

TOTAL: 60

PERIODS

OUTCOMES:

After studying this course, the student should be able to:

To be able to determine if a given system is linear/causal/stable

Capable of determining the frequency components present in a deterministic signal

Capable of characterizing LTI systems in the time domain and frequency domain

To be able to compute the output of an LTI system in the time and frequency domains

TEXT BOOKS:

Allan V.Oppenheim, S.Wilsky and S.H.Nawab, ―Signals and Systems‖, Pearson, 2015.(Unit 1-V)

REFERENCES

1 B. P. Lathi, ―Principles of Linear Systems and Signals‖, Second Edition, Oxford, 2009.
2. R.E.Zeimer, W.H.Tranter and R.D.Fannin, ―Signals & Systems - Continuous and Discrete‖, Pearson, 2007.
3. John Alan Stuller, ―An Introduction to Signals and Systems‖, Thomson, 2007.

UNIT - I

CLASSIFICATION OF SIGNALS AND SYSTEMS

SYLLABUS:

Standard signals - Step, Ramp, Pulse, Impulse, Real and complex exponentials and standards - classification of signals - Continuous time (CT) and Discrete Time (DT) signals, Periodic & Aperiodic signals, Deterministic & Random signals. Energy and power signals - classification of systems - CT systems and DT systems - Linear & Non linear, Time-Variant & Time-invariant, causal & Non-causal, stable & Unstable.

1.1. Standard signals:

Signal :-

→ Signal is one that carries information and is defined as a physical quantity that varies with one or more independent variable.

→ Example: Music, Speech.

Continuous time signal: (Analog signal)

→ A signal that is defined for every instants of time is known as continuous time signal.

→ Continuous time signals are continuous in amplitude and continuous in time.

→ It is denoted by $x(t)$.

Discrete - time signal :-

→ A signal that is defined for discrete instants of time is known as discrete time signal.

→ They are continuous in amplitude and discrete in time.

→ It is also obtained by sampling a continuous time signal.

→ It is denoted by $x(n)$.

Digital signal :-

→ The signals that are discrete in time and quantized in amplitude is called digital signal.

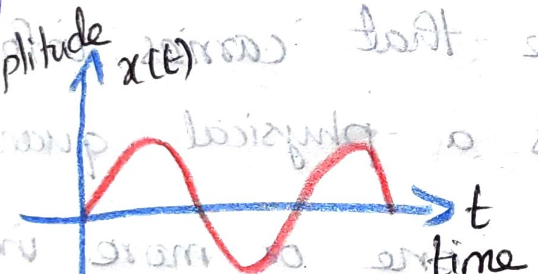


fig: Continuous time signal

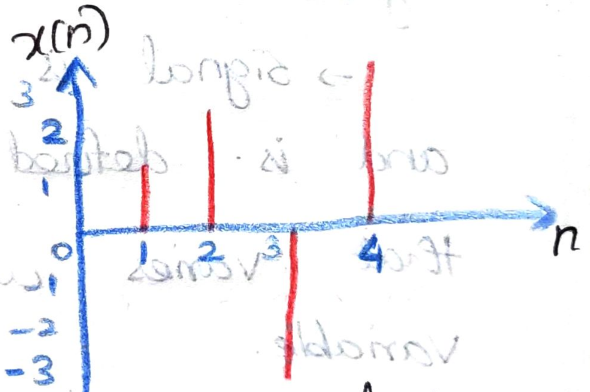


fig: Discrete time signal

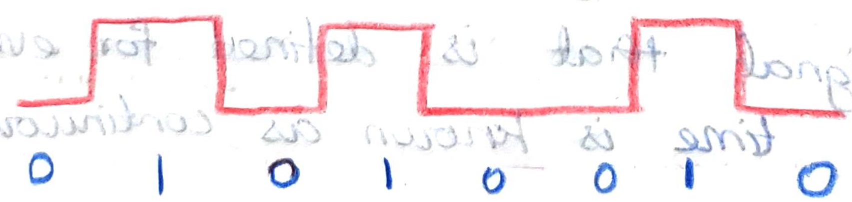


fig: Digital signal

1.1.1. Standard continuous time signals (Basic or Elementary)

(i) Step signal:

Unit step signal is defined as,

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0$$

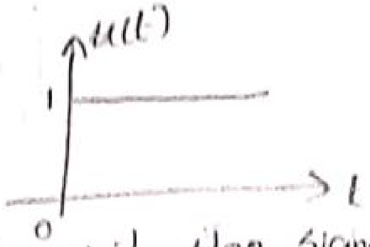
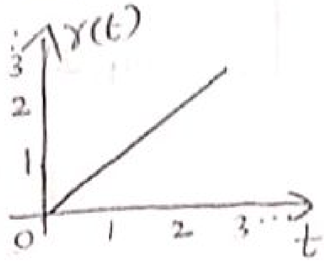


Fig: Unit step signal

(ii) Ramp signal:

Unit ramp signal is defined as,

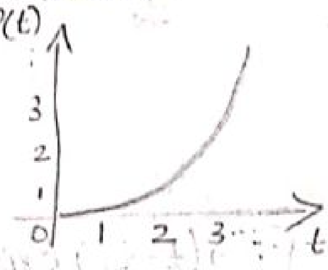
$$r(t) = t \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0$$



(iii) Parabolic signal

Unit parabolic signal is defined as,

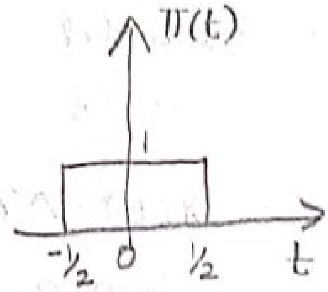
$$x(t) = \frac{t^2}{2} \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0$$



(iv) Unit pulse signal

It is defined as,

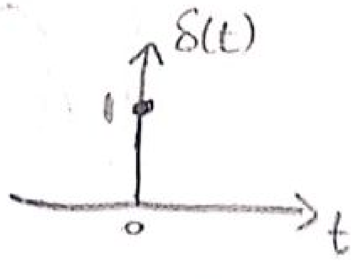
$$\Pi(t) = 1 \quad \text{for } |t| \leq 1/2$$
$$= 0 \quad \text{otherwise}$$



(v) Impulse signal:

It is defined as.

$$\delta(t) = 0 \quad \text{for } t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



Properties of Impulse signal:

Property 1:

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)}$$

Proof:

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)\,\delta(0) = x(0)$$

[$\because$ $\delta(t)$ exists only at $t=0$ and $\delta(0)=1$]

Property 2:

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)}$$

Proof:

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)\,\delta(t_0-t_0)$$

$$= x(t_0)\,\delta(0)$$

$$= x(t_0)$$

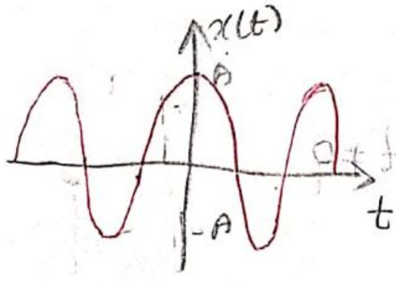
[$\because$ $\delta(t-t_0)$ exists only at $t=t_0$ and $\delta(0)=1$]

(vi) Sinusoidal Signal:

Cosinusoidal signal

$x(t) = A\cos(\Omega t + \varphi)$

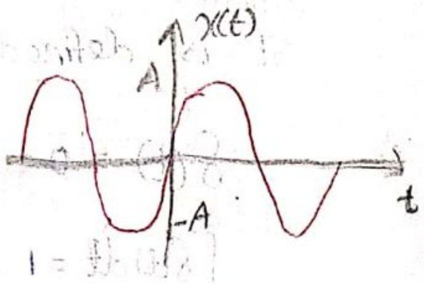
where $\Omega = 2\pi f = \frac{2\pi}{T}$ rad/sec



sinusoidal signal

$x(t) = A\sin(\Omega t + \varphi)$

$\Omega = 2\pi f = \frac{2\pi}{T}$ rad/sec



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(vii) Exponential signal:

a) Real Exponential Signal.

It is defined as $\boxed{x(t) = Ae^{at}}$

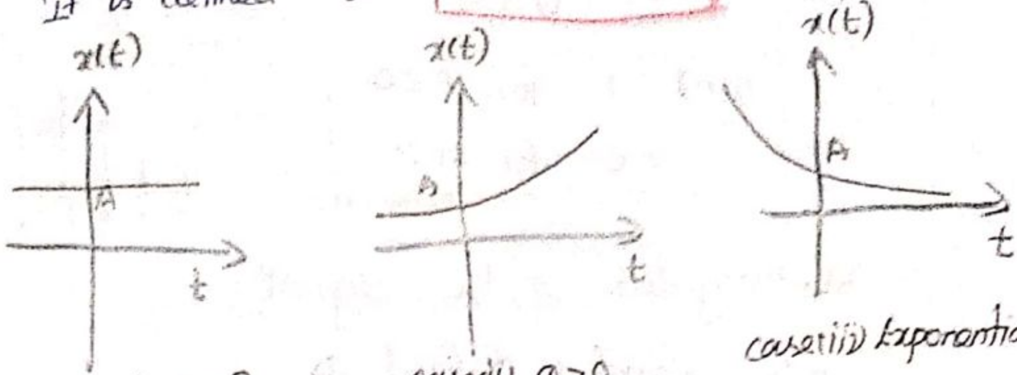


Fig: case(i) $a = 0$ DC signal

case(ii) $a > 0$ Exponentially growing signal

case(iii) Exponentially decaying signal

b) Complex exponential signal:

It is defined as,

$$\boxed{\begin{aligned} x(t) &= Ae^{st} = Ae^{(\sigma + j\Omega)t} = Ae^{\sigma t}e^{j\Omega t} \\ x(t) &= Ae^{\sigma t}(\cos\Omega t + j\sin\Omega t) \end{aligned}}$$

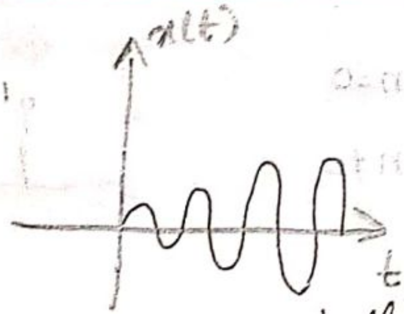


Fig: $\sigma > 0$ (Exponentially growing signal)

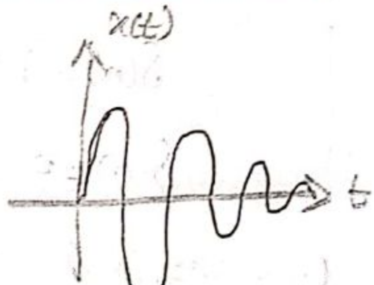


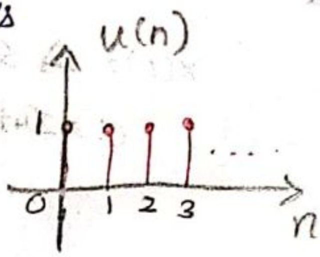
Fig: $\sigma < 0$ (exponentially decaying signal)

1.1.2. Basic (Elementary or standard) Discrete time signals:

(i) Step signal

Unit step signal is defined as

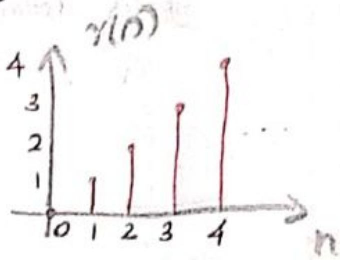
$$\boxed{u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}}$$



(ii) Ramp Signal :-

Unit ramp signal is defined as,

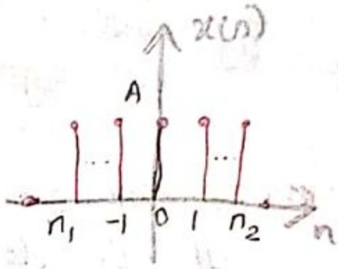
$$\boxed{\begin{aligned} r(n) &= 1 \quad \text{for } n \geq 0 \\ &= 0 \quad \text{for } n < 0 \end{aligned}}$$



(iii) Rectangular pulse signal:

Pulse signal is defined as,

$$\boxed{\begin{aligned} x(n) &= A \quad \text{for } n_1 \leq n \leq n_2 \\ &= 0 \quad \text{elsewhere} \end{aligned}}$$

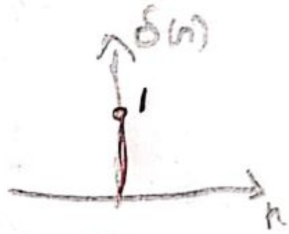


(iv) Unit Impulse signal:

Unit Impulse signal is defined as,

$$\delta(n) = 1 \quad \text{for } n = 0$$

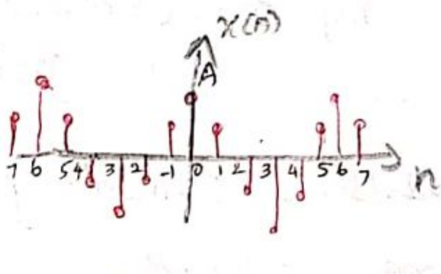
$$\delta(n) = 0 \quad \text{for } n \neq 0$$



(v) Sinusoidal signal:

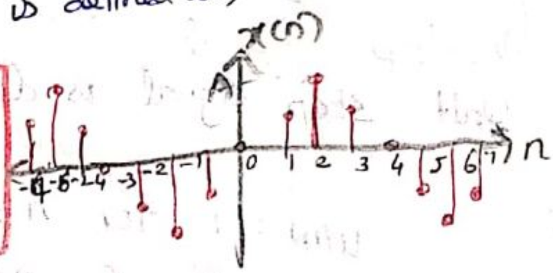
a) cosinusoidal signal is,

$$\boxed{\begin{aligned} x(n) &= A\cos(\omega n) \\ \omega &= 2\pi f = \frac{2\pi m}{N} \end{aligned}}$$



2) Sinusoidal signal is defined as,

$$\boxed{\begin{aligned} x(n) &= A\sin(\omega n) \\ \omega &= 2\pi f = \frac{2\pi m}{N} \end{aligned}}$$

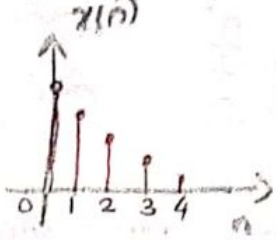


(vi) Exponential signal:

a) Real Exponential Signal:

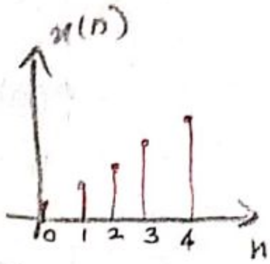
It is defined as,

$$\boxed{x(n) = a^n \quad \text{for } n \geq 0}$$



case (i) $0 < a < 1$

Decreasing exponential signal



case (ii) $a > 1$

Increasing exponential signal.

b) Complex Exponential Signal:

It is defined as,

$$x(n) = a^n e^{j\omega_0 n}$$

$$x(n) = a^n [\cos\omega_0 n + j\sin\omega_0 n]$$

$$x_r(n) = a^n \cos\omega_0 n \quad \& \quad x_i(n) = a^n \sin\omega_0 n.$$

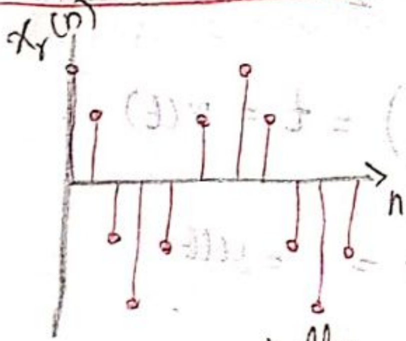


Fig: Exponentially decreasing cosine signal

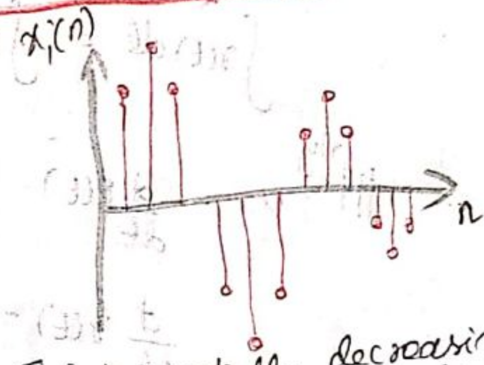


Fig: Exponentially decreasing sinusoidal signal

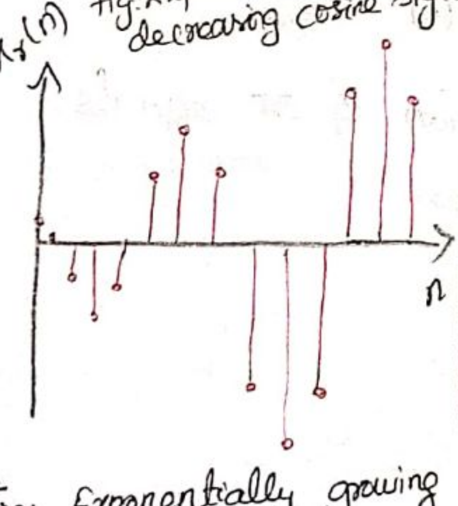


Fig: Exponentially growing co-sinusoidal signal

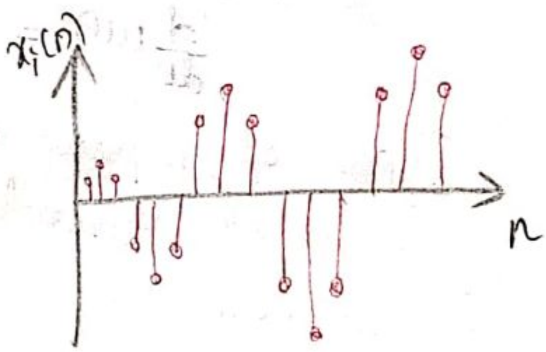
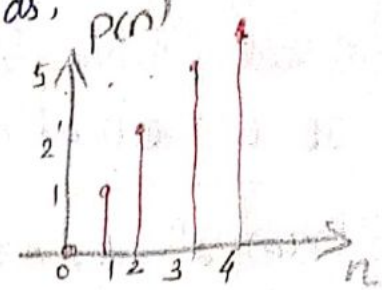


Fig: Exponentially growing Sinusoidal Signal

(viii) Parabolic signal:

Unit parabolic signal is defined as,

$$\boxed{p(n) = \begin{cases} \frac{n^2}{2} & n \geq 0 \\ 0 & n < 0 \end{cases}}$$



Relationship b/n signals:-

$$\delta(t) \xrightarrow{\int} u(t) \xrightarrow{\int} r(t) \xrightarrow{\int} p(t)$$

$$p(t) \xrightarrow{d/dt} r(t) \xrightarrow{d/dt} u(t) \xrightarrow{d/dt} \delta(t)$$

ie)

$$\int \delta(t)\, dt = 1 = u(t)$$

$$\int u(t)\, dt = \int dt = t = r(t)$$

$$\int r(t)\, dt = \int t\, dt = \frac{t^2}{2} = p(t)$$

|||^{rly},

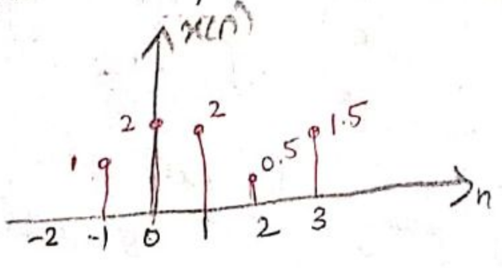
$$\frac{d}{dt} p(t) = \frac{d}{dt}\left(\frac{t^2}{2}\right) = t = r(t)$$

$$\frac{d}{dt} r(t) = \frac{dt}{dt} = 1 = u(t)$$

$$\frac{d}{dt} u(t) = \delta(t)$$

Types of Representation of DT signals:-

(a) Graphical Representation:-



(b) Functional Representation :-

$$x(n) = \begin{cases} 1 & \text{for } n=-1 \\ 2 & \text{for } n=0,1 \\ 0.5 & \text{for } n=2 \\ 1.5 & \text{for } n=3 \\ 0 & \text{Otherwise.} \end{cases}$$

(c) Tabular Representation:-

n	-1	0	1	2	3
$x(n)$	1	2	2	0.5	1.5

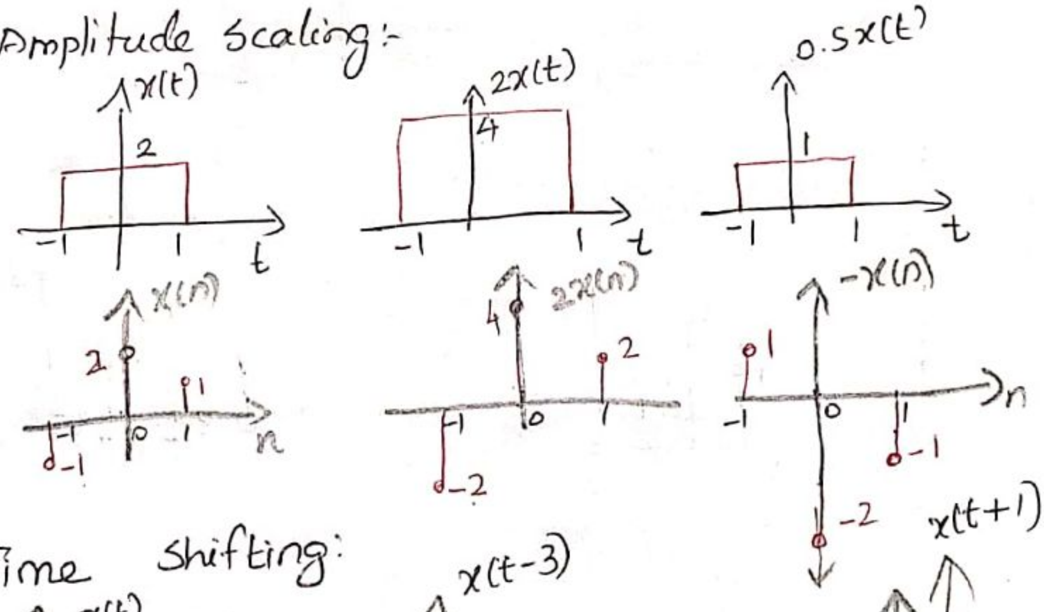
(d) sequence representation:-

$$x(n) = \{1, \underset{\uparrow}{2}, 2, 0.5, 1.5\}$$

$\uparrow$ - indicates $n=0$.

Mathematical operations on signals:-

(i) Amplitude scaling:-



(ii) Time shifting:

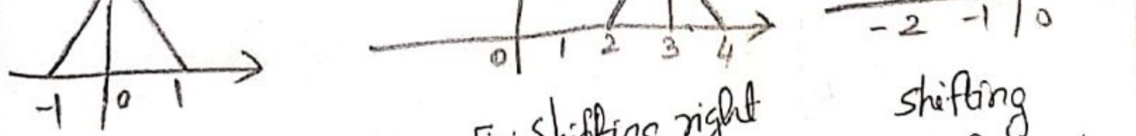


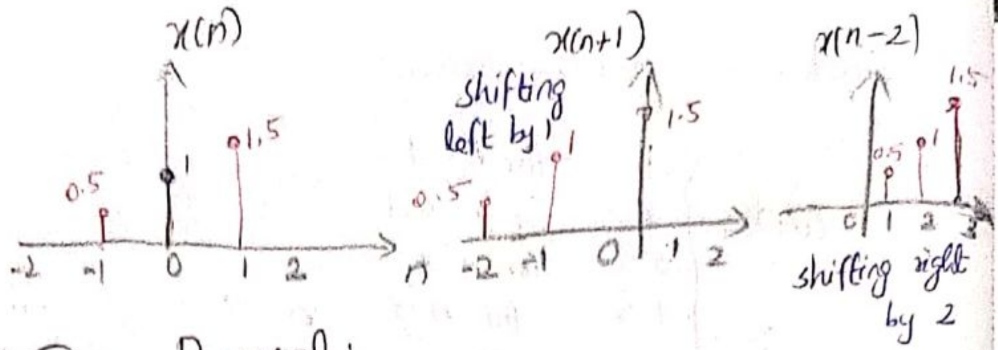
Fig: Shifting right by 3

shifting left by 1

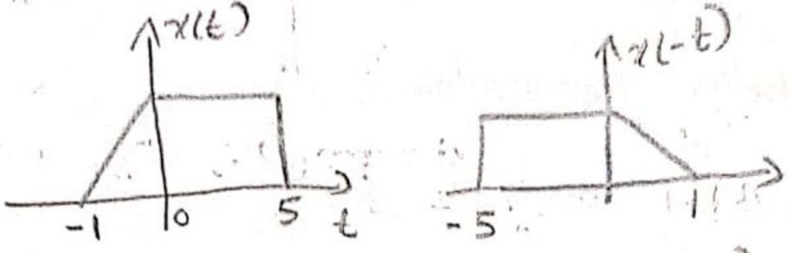
Hint:

$\left.\begin{matrix} - - \\ + + \end{matrix}\right\} \Rightarrow$ left shift $\leftarrow$

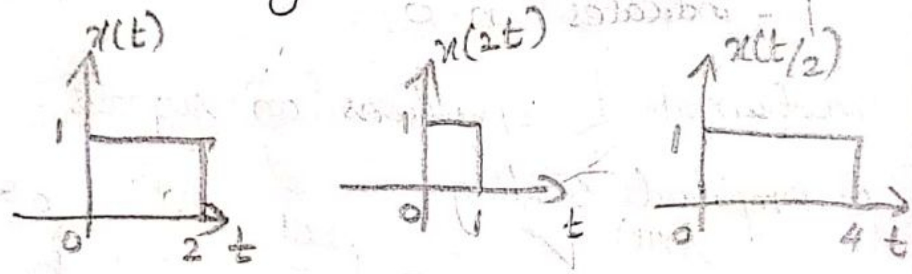
$(+-)$ or $(-+)$ $\Rightarrow$ Right shift $\rightarrow$

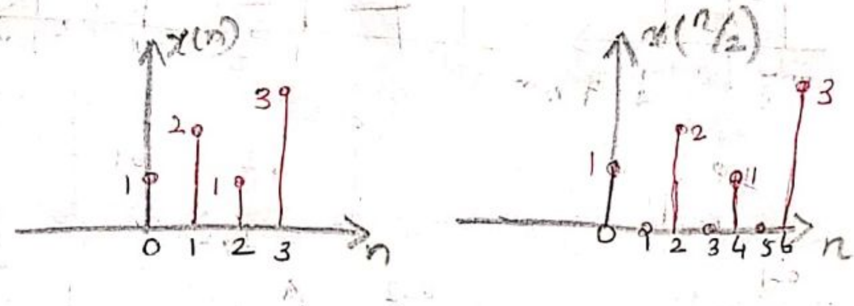


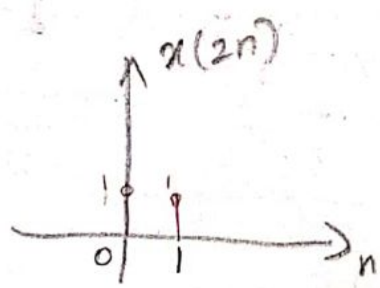
(iii) Time Reversal:



(iv) Time scaling:



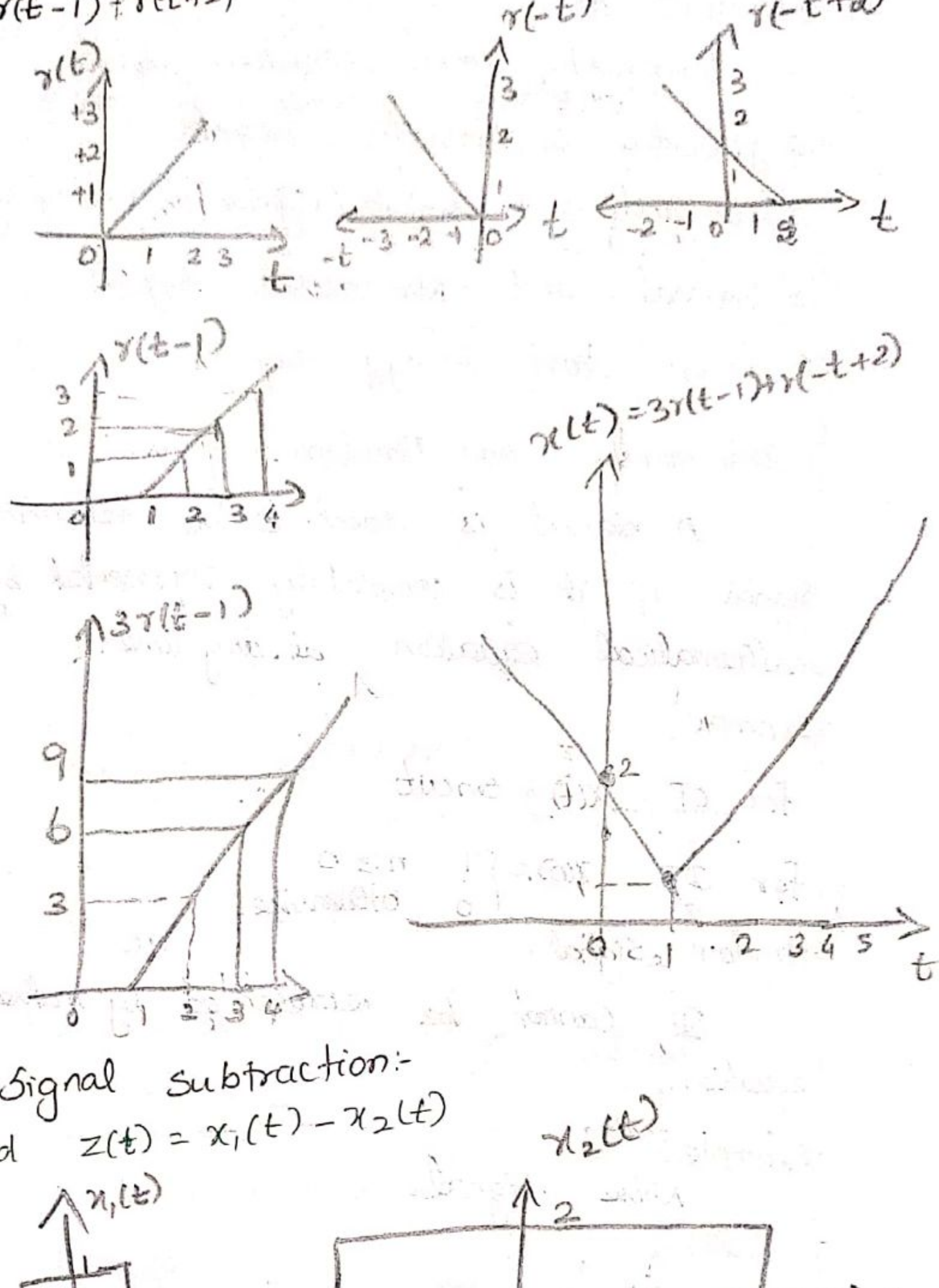




(v) Signal Addition:-

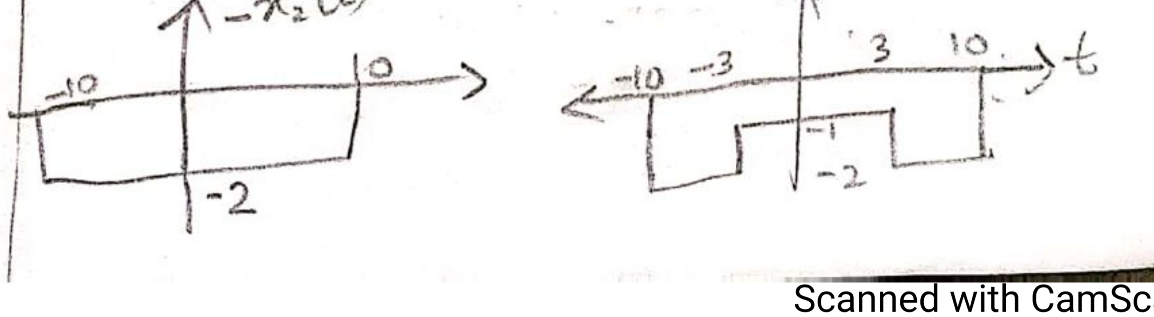
Sketch $x(t) = 3r(t-1) + r(-t+2)$

$3r(t-1) + r(-t+2)$



(vi) Signal subtraction:-

Find $z(t) = x_1(t) - x_2(t)$



1.2. Classification of signals:

Both CT and DT signals are further classified as.

1. Deterministic and Random signals.
2. Periodic & Aperiodic signals
3. Even and Odd signals (Symmetric & Antisymmetric signals)
4. Causal and Non causal signals
5. Power and Energy signals.

1. Deterministic and Random signals:-

A signal is said to be deterministic signal if it is completely represented by mathematical equation at any time.

Example:-

for CT $x(t) = \sin\omega t$

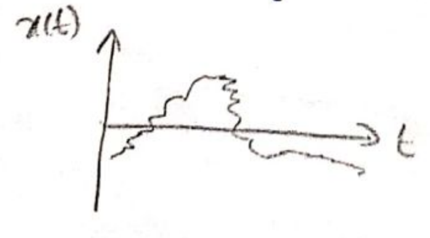
for DT $x(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

Random signal:

It cannot be represented by mathematical equation.

Example:-

Noise signal



2. Periodic and Aperiodic signals

A signal is said to be periodic, if it repeats at regular interval of time.

Condition for periodic signal:

$$\boxed{\begin{array}{l} \text{CT signal}: x(t) = x(t+T) \\ \text{DT signal}: x(n) = x(n+N) \end{array}}$$

$$T = \frac{2\pi}{\Omega} \qquad N = \frac{2\pi k}{\omega}$$

Example:

Ex.1.2.1. Test the periodicity:

(a) $x(t) = \sin t$

$$x(t) = \sin t$$

$$x(t+T) = \sin(t+T)$$

$$= \sin(t+2\pi)$$

$$= \sin t \cos 2\pi + \cos t \sin 2\pi$$

$$= \sin t$$

$$= x(t)$$

$$x(t+T) = x(t)$$

It is the periodic signal.

(b) $x(t) = \sin 20\pi t + \sin 20t$

$$T_1 = \frac{2\pi}{\Omega_1} = \frac{2\pi}{20\pi} = \frac{1}{10}$$

$$T_2 = \frac{2\pi}{\Omega_2} = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$\frac{T_1}{T_2} = \frac{1}{10} \times \frac{10}{\pi} = \frac{1}{\pi} \text{ . Irrational number}$$

So the signal is Aperiodic signal.

(c) $x(n) = 4e^{j\left(\frac{2\left(n+\frac{1}{2}\right)}{5}\right)}$

$\omega = \frac{2}{5}$ (co-efficient of 'n' is ω)

$N = \frac{2\pi}{\omega}k = \frac{2\pi}{(2/5)}k = 5\pi k$

For any value of k, N is not an integer.

∴ The given signal is Aperiodic.

(d) $x(n) = \cos\left(\frac{2\pi n}{3}\right) + \cos\left(\frac{2\pi n}{5}\right)$

$N_1 = \frac{2\pi}{\omega_1}k = \frac{2\pi}{\frac{2\pi}{3}}k = 3k$, $N_2 = \frac{2\pi}{\omega_2}k$

If k is 1, N_1 is integer

$N_1 = 3$

$N_2 = \frac{2\pi}{\frac{2\pi}{5}}k$

$N_2 = 5k$

if $k=1$, N_2 is integer

$N_2 = 5$

$\frac{N_1}{N_2} = \frac{3k}{5k} = \frac{3}{5}$

It is rational number, Hence the given signal is periodic.

Period $N = 5N_1 = 3N_2 = 5\times3 = 3\times5 = 15$.

Fundamental period $N = 15$

3. Even and odd signals :-

A signal is said to be even if satisfy the following Condition.

$$\boxed{\begin{array}{ll} \text{For CT} & x(-t) = x(t) \\ \text{For DT} & x(-n) = x(n) \end{array}}$$

A signal is said to be odd if it satisfy the following condition.

$$\boxed{\begin{array}{ll} \text{For CT} & x(-t) = -x(t) \\ \text{for DT} & x(-n) = -x(n) \end{array}}$$

For CT even component $\boxed{x_e(t) = \frac{x(t)+x(-t)}{2}}$

odd component $\boxed{x_o(t) = \frac{x(t)-x(-t)}{2}}$

For DT

$$\boxed{\begin{array}{ll} \text{even component} & x_e(n) = \frac{x(n)+x(-n)}{2} \\ \text{odd component} & x_o(n) = \frac{x(n)-x(-n)}{2} \end{array}}$$

Example:

Find the even and odd component of the signal:-

(a) $x(t) = \cos t \sin t + 2\sin t + \cos^2 t \sin t + \cos t$

$x(t) = \cos t \sin t + 2\sin t + \cos^2 t \sin t + \cos t$

Put $t = -t$,

$$x(-t) = \cos(-t)\sin(-t) + 2\sin(-t) + \cos^2(-t)\sin(-t) + \cos(-t)$$

$$x(-t) = -\cos t \sin t - 2\sin t - \cos^2 t \sin t + \cos t$$

$(\because \cos(-t) = \cos t$
$\sin(-t) = -\sin t)$

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{2\cos t}{2} = \cos t$$

$$x_e(t) = \cos t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$x_o(t) = \cos t \sin t + 2\sin t + \cos^2 t \sin t.$$

(b) $x(n) = \{2, 1, \underset{\uparrow}{3}, -2, 4\}$

$x(-2) \quad x(-1) \quad x(0) \quad x(1) \quad x(2)$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_e(0) = \frac{x(0) + x(-0)}{2}$$

$$= \frac{1}{2}(3+3)$$

$$x_e(0) = 3$$

$$x_e(1) = \frac{x(1) + x(-1)}{2}$$

$$x_e(1) = \frac{-2+1}{2} = \frac{-1}{2} = -0.5$$

$$x_e(2) = \frac{x(2) + x(-2)}{2} = \frac{4+2}{2} = 3$$

$$x_e(-1) = \frac{x(-1) + x(1)}{2} = \frac{1-2}{2} = -0.5$$

$$x_e(-2) = \frac{x(-2) + x(2)}{2} = \frac{2+4}{2} = 3$$

$x_e(n) = \{3, -0.5, \underset{\uparrow}{3}, -0.5, 3\}$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$x_o(0) = \frac{x(0) - x(-0)}{2} = \frac{3-3}{2} = 0$$

$$x_o(1) = \frac{x(1) - x(-1)}{2} = \frac{-2-1}{2} = \frac{-3}{2} = -1.5$$

$$x_o(2) = \frac{x(2) - x(-2)}{2} = \frac{4-2}{2} = \frac{2}{2} = 1$$

$$x_o(-1) = \frac{x(-1) - x(1)}{2} = \frac{1-(-2)}{2} = \frac{3}{2} = 1.5$$

$$x_o(-2) = \frac{x(-2) - x(2)}{2} = \frac{2-4}{2} = \frac{-2}{2} = -1$$

$$x_o(n) = \{-1, 1.5, \underset{\uparrow}{0}, -1.5, 1\}$$

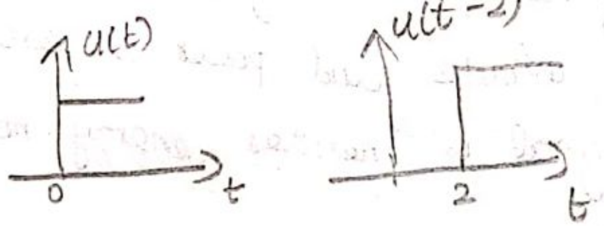
4. Causal and Noncausal signals :-

The continuous time signal is said to be causal if $x(t) = 0$ for $t<0$ otherwise the signal is said to non causal.

The DT signal is said to be causal if the signal $x(n) = 0$ for $n<0$. otherwise the signal is said to be noncausal.

Example :-

a) $x(t) = e^{-2t} u(t-2)$

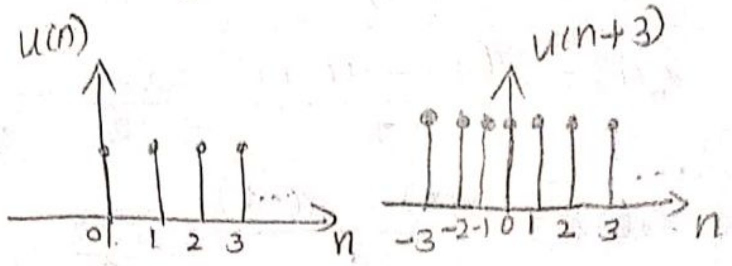


e^{-2t} is multiplied by $u(t-2)$.

$$u(t-2) = \begin{cases} 1 & t \geq 2 \\ 0 & t < 2 \end{cases}$$

$\therefore$ The given signal is causal.

b) $x(n) = \left(\frac{1}{2}\right)^n u(n+3)$



$$u(n+3) = \begin{cases} 1 & n \le -3 \\ 0 & \text{otherwise.} \end{cases}$$

$\left(\frac{1}{2}\right)^n$ is multiplied by $u(n+3)$,

$\therefore x(n) \neq 0$ for $n < 0$

$\therefore$ Therefore the signal is non-causal signal.

5. Energy and power signal :-

Energy signal :-

Total energy is finite and average Power is zero, then the signal is said to be energy signal.

Power Signal :-

The total energy is infinite and average power is finite, then the signal is said to be power signal.

Neither Energy nor power signal :-

Energy is infinite and power is zero means the signal is neither energy nor power signal.

CT Signal :

$$\text{Energy } E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Average Power $P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

DT Signal:

Energy $E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$

Average Power $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

Check whether the given signal is power or energy.

a) $x(t) = e^{\alpha t} u(-t)$

$$\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt} = \int_{-\infty}^{\infty} |e^{\alpha t} u(-t)|^2 \, dt$$

$$\boxed{\therefore u(-t) = \begin{cases} 0 & t > 0 \\ 1 & t < 0 \end{cases}}$$

$$E = \int_{-\infty}^{0} |e^{\alpha t}|^2 \, dt$$

$$= \int_{-\infty}^{0} e^{2\alpha t} \, dt = \left[\frac{e^{2\alpha t}}{2\alpha} \right]_{-\infty}^{0}$$

$$E = \frac{1}{2\alpha} \Rightarrow \boxed{\text{finite}}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{\alpha t} \cdot u(-t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{0} |e^{\alpha t}|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{e^{2\alpha t}}{2\alpha}\right]_{-T}^{0}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \cdot \frac{1}{2\alpha} \left[e^{0} - e^{-2\alpha T}\right]$$

$$= \lim_{T\to\infty} \frac{1}{4\alpha T}\left[1 - e^{-2\alpha T}\right]$$

$$= \frac{1}{\infty}\left[1 - e^{-\infty}\right] = \frac{1}{\infty}\left[1 - 0\right]$$

$$= \frac{1}{\infty} = 0$$

$$\boxed{P = 0}$$

Energy is finite and Power is Zero. Therefore the given signal is energy signal.

b) $x(n) = e^{j(\pi/3 n + \pi/10)}$

$$\boxed{\text{Energy } E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2}$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|e^{j(\pi/3 n + \pi/10)}\right|^2$$

$(\because |z|^2 = z z^*)$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} e^{j(\pi/3 n + \pi/10)} \cdot e^{-j(\pi/3 n + \pi/10)}$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} e^{j(\pi/3 n + \pi/10 - \pi/3 n - \pi/10)}$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} 1 \quad = \lim_{N\to\infty} 2N+1 \Rightarrow \text{Infinite}$$

$$E = \infty$$

Scanned with CamScanner

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j(\pi/3\, n + \pi/10)} \right|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \, 2N+1$$

$$= \lim_{N \to \infty} 1$$

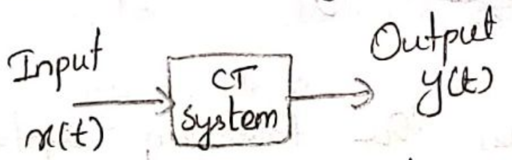
$P = 1 \Rightarrow$ Finite

Energy is infinite and power is finite. Therefore the given signal is power signal.

System:-

It is an interconnection of functional blocks. It produces output with respect to an input signal.

Continuous time systems:-

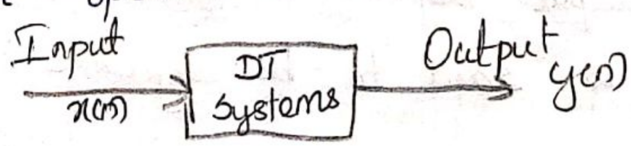


If the system operates on continuous time signals. Then it is called continuous time system.

Example:- Amplifiers, Analog transmitter and receiver.

Discrete time systems:-

It operates on discrete time signals.



Example:- Microprocessor, Computer.

classification of continuous time and discrete time systems

1. static and dynamic system
2. Linear and non linear system.
3. Time Variant and time invariant system.
4. Causal and non causal system.
5. Stable and unstable system.

1. static and dynamic system (system with memory or memory less system).

If the output depends only on Present input and it does not depends on past or future inputs. Then it is called static system.

If the output depends on present, Past input or future outputs then the system is called Dynamic system.

Example:

Check whether the following systems are Static or dynamic.

(a) $y(t) = x(t-3)$

At $t=0$, $y(0) = x(-3)$

$\therefore$ The output depends on past input, so the system is said to be Dynamic

b) $y(t) = x^2(t)$

At $t=0$, $y(0) = x^2(0)$

The output depends only on the present input so the system is called static.

(c) $y(t) = \frac{d}{dt} x(t)$

Differention makes system dynamic. Hence the system is dynamic system

(d) $y(n) = x(n) \cos(\omega n)$

At $n=0$, $y(0) = x(0) \cos(0)$

The output depends on present input. Hence the given system is static system.

(e) $y(n) = x(n^3)$

At $n=0$, $y(0) = x(0)$

$n=2$, $y(2) = x(2^3) = x(8)$

The output depends on present and future inputs. Hence the system is Dynamic system.

2. Linear and non-linear system:

If the system obeys superposition principle then the system is said to be Linear system.

For CT system,

$$\boxed{\begin{aligned} y_3(t) &= H[a_1 x_1(t) + a_2 x_2(t)] = a_1 H[x_1(t)] + a_2 H[x_2(t)] \\ y_3(t) &= a_1 y_1(t) + a_2 y_2(t) \end{aligned}}$$

$H[x(t)] = y(t)$ is the response of continuous time system to the input $x(t)$.

For DT, $\boxed{H[a_1 x_1(n) + a_2 x_2(n)] = a_1 H[x_1(n)] + a_2[H[x_2(n)]]}$

Example:
Check whether the following systems are linear or not.

a) $y(t) = x^2(t)$

$$y_1(t) = x_1^2(t)$$

$$y_2(t) = x_2^2(t)$$

$$y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t) \quad \text{--- (1)}$$

$$y_3(t) = (x_1(t) + x_2(t))^2 = x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t) \quad \text{--- (2)}$$

$$(1) \neq (2)$$

ie) $y_3(t) \neq y_1(t) + y_2(t)$

Hence the system is nonlinear

b) $\dfrac{dy(t)}{dt} + 5y(t) = x(t)$

This is the linear differential equation. Hence the system is linear.

$$\frac{dy_1(t)}{dt} + 5y_1(t) = x_1(t)$$

$$\frac{dy_2(t)}{dt} + 5y_2(t) = x_2(t)$$

$$\frac{dy_1(t)}{dt} + 5y_1(t) + \frac{dy_2(t)}{dt} + 5y_2(t) = x_1(t) + x_2(t) \quad \text{--- (1)}$$

$$\frac{d}{dt}(y_1(t) + y_2(t)) + 5[y_1(t) + y_2(t)] = x_1(t) + x_2(t)$$

$$\frac{d}{dt}y_1(t) + \frac{d}{dt}y_2(t) = 5y_1(t) + 5y_2(t) = x_1(t) + x_2(t) \quad \text{--- (2)}$$

$$(1) = (2)$$

Hence the system is linear.

(c) $y(n) = ax(n) + b$

$$y_1(n) = ax_1(n) + b$$

$$y_2(n) = ax_2(n) + b$$

$$y_1(n) + y_2(n) = ax_1(n) + b + ax_2(n) + b \quad \text{--- (1)}$$

$$y_1(n) + y_2(n) = ax_1(n) + ax_2(n) + 2b$$

$$y_3(n) = a[x_1(n) + x_2(n)] + b$$

$$y_3(n) = ax_1(n) + ax_2(n) + b \quad \text{--- (2)}$$

$$(1) \neq (2)$$

Hence the system is non-linear.

(d) $y(n) = x(n)\cos(\omega_0 n)$

$$y_1(n) = x_1(n)\cos(\omega_0 n)$$

$$y_2(n) = x_2(n)\cos(\omega_0 n)$$

$$y_1(n) + y_2(n) = x_1(n)\cos(\omega_0 n) + x_2(n)\cos(\omega_0 n) \quad \text{--- (1)}$$

$$y_3(n) = [x_1(n) + x_2(n)]\cos(\omega_0 n)$$

$$y_3(n) = x_1(n)\cos(\omega_0 n) + x_2(n)\cos\omega_0 n \quad \text{--- (2)}$$

$$(1) = (2)$$

Hence the system is linear.

3. Time Variant and Invariant System:-

A system is time invariant if the timeshift in the input signal results in corresponding time shift in the output

For

$$\boxed{\begin{array}{ll} \text{CT system:} & f[x(t-t_1)] \neq y(t-t_1) \\ \text{DT system:} & f[x(n-k)] \neq y(n-k) \end{array}}$$

Then the system is called Time Variant

Example:

Check the following systems are Time-Invariant or not.

(a) $y(t) = \sin x(t)$

Output of the system for the delayed input,

$$y(t, t_1) = \sin x(t - t_1)$$

Delay the output by t_1,

$$y(t - t_1) = \sin x(t - t_1)$$

$$y(t, t_1) = y(t - t_1)$$

Hence the system is time invariant.

b) $y(t) = x(t) \cos 200\pi t$

The output of the system for the delayed input is,

$$y(t, t_1) = x(t - t_1) \cos 200\pi t \quad -(1)$$

Delay the output $y(t)$ by t_1

$$y(t - t_1) = x(t - t_1) \cos 200\pi (t - t_1) \quad -(2)$$

$$y(t, t_1) \neq y(t - t_1)$$

Hence the given system is time invariant

(c) $y(n) = x(2n)$

Output of the system for the delayed input is,

$$y(n, k) = x(2n - k)$$

Delay the output by $n-k$,

$$y(n-k) = x[2(n-k)]$$

$$y(n, k) \neq y(n-k)$$

Hence the given system is time Variant (or) shift variant.

(d) $y(n) = ax(n) + b$.

Output of the system for the delayed input,

$$y(n, k) = a x(n-k) + b$$

Delay the output by $n-k$,

$$y(n-k) = ax(n-k) + b$$

$$y(n, k) = y(n-k)$$

Hence the system is time invariant (or) shift invariant.

4. Causal and Non Causal System

Causal System

The output depends on present and past inputs. It does not depends on future input.

Non causal System:-

If the output depends on future inputs also then the system is said to be non causal.

Examples:

check whether the following systems are causal or Non-Causal:

(a) $y(t) = x(t)\,x(t-1)$

sub $t=0$, $y(0) = x(0)\,x(-1)$

$t=1$ $y(1) = x(1)\,x(0)$

$t=-1$ $y(-1) = x(-1)\,x(-2)$

$\therefore$ The output depends on present and past inputs. Therefore the system is causal.

(b) $y(t) = x(t-3) + x(3-t)$

when $t=0$, $y(0) = x(-3) + x(3)$

The output depends on future input.

So the system is non-causal.

(c) $y(n) = x(2n)$

sub $n=0$; $y(0) = x(0)$

sub $n=1$, $y(1) = x(2)$

The output depends on the future input. Therefore the system is non causal.

(d) $y(n) = x(n-1) + \frac{1}{x(n)}$

sub $n=0$, $y(0) = x(-1) + \frac{1}{x(0)}$

$y(1) = x(0) + \frac{1}{x(1)}$

$y(-1) = x(-2) + \frac{1}{x(-1)}$

$\therefore$ The output depends on present and Past inputs. Therefore the system is causal system.

5. stable and unstable systems:-

The system that produces bounded output for bounded input is said to be stable system.

It is generally called as BIBO (Bounded Input Bounded Output) Stable system.

If the system gives unbounded output for bounded input is called unstable system.

$$y_1(n) + y_2(n) = x_1(n)\cos(\omega_0 n) + x_2(n)\cos(\omega_0 n) \quad \text{——}(1)$$

$$y_3(n) = [x_1(n) + x_2(n)]\cos(\omega_0 n)$$

$$y_3(n) = x_1(n)\cos(\omega_0 n) + x_2(n)\cos\omega_0 n \quad \text{——}(2)$$

(1) = (2)

Hence the system is linear.

3. Time Variant and Invariant System:-

A system is time invariant if the timeshift in the input signal results in corresponding time shift in the output

For

$$\boxed{\begin{array}{l} \text{CT system: } f[x(t-t_1)] \neq y(t-t_1) \\ \text{DT system: } f[x(n-k)] \neq y(n-k) \end{array}}$$

Then the system is called Time Variant

Example:

Check the following systems are Time-Invariant or not.

(a) $y(t) = \sin x(t)$

Output of the system for the delayed input,

$$y(t, t_1) = \sin x(t - t_1)$$

Delay the output by t_1,

$$y(t - t_1) = \sin x(t - t_1)$$

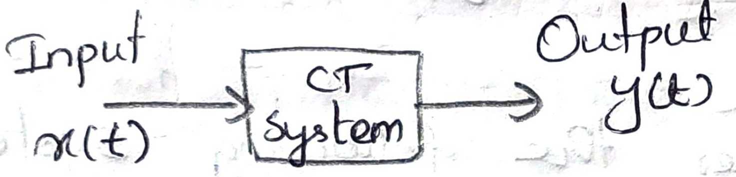
$$y(t, t_1) = y(t - t_1)$$

Hence the system is time invariant.

System:-

It is an interconnection of functional blocks. It produces output with respect to an input signal.

Continuous time systems:-



If the system operates on continuous time signals. Then it is called continuous time system.

Example:- Amplifiers, Analog transmitter and receiver.

Discrete time systems:-

It operates on discrete time signals.

Example:- Microprocessor, Computer.

classification of continuous time and discrete time systems.

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2. Linear and non linear system.
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5. Stable and unstable system.

1. static and dynamic system (system with memory or memory less system).

If the output depends only on Present input and it does not depends on past or future inputs. Then it is called static system.

If the output depends on present, Past input or future outputs then the system is called Dynamic system.

Example:

Check whether the following systems are Static or dynamic.

(a) $y(t) = x(t-3)$

At $t=0$, $y(0) = x(-3)$

∴ The output depends on past input, so the system is said to be Dynamic

b) $y(t) = x^2(t)$

At $t=0$, $y(0) = x^2(0)$

The output depends only on the present input so the system is called static.

(c) $y(t) = \frac{d}{dt}x(t)$

Differentiation makes system dynamic.
Hence the system is dynamic system.

(d) $y(n) = x(n)\cos(\omega_0 n)$

At $n=0$, $y(0) = x(0)\cos(0)$

The output depends on present input.
Hence the given system is static system.

(e) $y(n) = x(n^3)$

At $n=0$, $y(0) = x(0)$

$n=2$, $y(2) = x(2^3) = x(8)$

The output depends on present and future inputs. Hence the system is Dynamic system.

2. Linear and non-linear System:

If the system obeys superposition principle then the system is said to be Linear system.

For CT system,

$$\boxed{\begin{gathered} y_3(t) = H[a_1x_1(t) + a_2x_2(t)] = a_1H[x_1(t)] + a_2H[x_2(t)] \\ y_3(t) = a_1y_1(t) + a_2y_2(t) \end{gathered}}$$

$H[x(t)] = y(t)$ is the response of continuous time system to the input $x(t)$.

For DT, $\boxed{H[a_1 x_1(n) + a_2 x_2(n)] = a_1 H[x_1(n)] + a_2[H[x_2(n)]]}$

Example:-
check whether the following systems are linear or not.

a) $y(t) = x^2(t)$

$$y_1(t) = x_1^2(t)$$

$$y_2(t) = x_2^2(t)$$

$$y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t) \quad \text{— (1)}$$

$$y_3(t) = (x_1(t) + x_2(t))^2 = x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t) \quad \text{— (2)}$$

$$(1) \neq (2)$$

i.e) $y_3(t) \neq y_1(t) + y_2(t)$

Hence the system is nonlinear

b) $\dfrac{dy(t)}{dt} + 5y(t) = x(t)$

This is the linear differential equation. Hence the system is linear.

$$\frac{dy_1(t)}{dt} + 5y_1(t) = x_1(t)$$

$$\frac{dy_2(t)}{dt} + 5y_2(t) = x_2(t)$$

$$\frac{dy_1(t)}{dt} + 5y_1(t) + \frac{dy_2(t)}{dt} + 5y_2(t) = x_1(t) + x_2(t) \quad \text{--- (1)}$$

$$\frac{d}{dt}(y_1(t) + y_2(t)) + 5[y_1(t) + y_2(t)] = x_1(t) + x_2(t)$$

$$\frac{d}{dt}y_1(t) + \frac{d}{dt}y_2(t) = 5y_1(t) + 5y_2(t) = x_1(t) + x_2(t) \quad \text{--- (2)}$$

$$(1) = (2)$$

Hence the system is linear.

(c) $y(n) = ax(n) + b$

$$y_1(n) = ax_1(n) + b$$

$$y_2(n) = ax_2(n) + b$$

$$y_1(n) + y_2(n) = ax_1(n) + b + ax_2(n) + b \quad \text{--- (1)}$$

$$y_1(n) + y_2(n) = ax_1(n) + ax_2(n) + 2b$$

$$y_3(n) = a[x_1(n) + x_2(n)] + b$$

$$y_3(n) = ax_1(n) + ax_2(n) + b \quad \text{--- (2)}$$

$$(1) \neq (2)$$

Hence the system is non-linear.

(d) $y(n) = x(n)\cos(\omega_0 n)$

$$y_1(n) = x_1(n)\cos(\omega_0 n)$$

$$y_2(n) = x_2(n)\cos(\omega_0 n)$$

$$y_1(n) + y_2(n) = x_1(n)\cos(\omega_0 n) + x_2(n)\cos(\omega_0 n) \quad \text{--- (1)}$$

$$y_3(n) = [x_1(n) + x_2(n)]\cos(\omega_0 n)$$

$$y_3(n) = x_1(n)\cos(\omega_0 n) + x_2(n)\cos\omega_0 n \quad \text{--- (2)}$$

$(1) = (2)$

Hence the system is linear.

3. Time Variant and Invariant System:-

A system is time invariant if the time shift in the input signal results in corresponding time shift in the output

For

$$\boxed{\begin{array}{ll} \text{CT system:} & f[x(t-t_1)] \neq y(t-t_1) \\ \text{DT system:} & f[x(n-k)] \neq y(n-k) \end{array}}$$

Then the system is called Time Variant

Example:

Check the following systems are Time-Invariant or not.

(a) $y(t) = \sin x(t)$

Output of the system for the delayed input,

$$y(t, t_1) = \sin x(t - t_1)$$

Delay the output by t_1,

$$y(t - t_1) = \sin x(t - t_1)$$

$$y(t, t_1) = y(t - t_1)$$

Hence the system is time invariant.

b) $y(t) = x(t)\cos 200\pi t$

The output of the system for the delayed input is,

$$y(t, t_1) = x(t-t_1)\cos 200\pi t \quad \text{—(1)}$$

Delay the output $y(t)$ by t_1

$$y(t-t_1) = x(t-t_1)\cos 200\pi(t-t_1) \quad \text{—(2)}$$

$$y(t, t_1) \neq y(t-t_1)$$

Hence the given system is time invariant.

(c) $y(n) = x(2n)$

Output of the system for the delayed input is,

$$y(n, k) = x(2n-k)$$

Delay the output by $n-k$,

$$y(n-k) = x[2(n-k)]$$

$$y(n,k) \neq y(n-k)$$

Hence the given system is time variant (or) shift variant.

(d) $y(n) = ax(n) + b$.

Output of the system for the delayed input,

$$y(n,k) = a\,x(n-k) + b$$

Delay the output by $n-k$,

$$y(n-k) = ax(n-k) + b$$

$$y(n,k) = y(n-k)$$

Hence the system is time invariant (or) shift invariant.

4. Causal and Non Causal System

Causal System

The output depends on present and past inputs. It does not depends on future input.

Non causal System:-

If the output depends on future inputs also then the system is said to be non causal.

Examples:-

check Whether the following systems are causal or Non-Causal:

(a) $y(t) = x(t)\,x(t-1)$

sub $t=0$, $y(0) = x(0)\,x(-1)$

$t=1$, $y(1) = x(1)\,x(0)$

$t=-1$ $y(-1) = x(-1)\,x(-2)$

∴ The output depends on present and past inputs. Therefore the system is causal.

(b) $y(t) = x(t-3) + x(3-t)$

when $t=0$, $y(0) = x(-3) + x(3)$

The output depends on future input.

So the system is non-causal.

(c) $y(n) = x(2n)$

sub $n=0$, $y(0) = x(0)$

sub $n=1$, $y(1) = x(2)$

The output depends on the future input. Therefore the system is non causal.

(d) $y(n) = x(n-1) + \frac{1}{x(n)}$

sub $n=0$, $y(0) = x(-1) + \frac{1}{x(0)}$

$y(1) = x(0) + \frac{1}{x(1)}$

$y(-1) = x(-2) + \frac{1}{x(-1)}$

$\therefore$ The output depends on present and Past inputs. Therefore the system is causal system.

5. stable and Unstable systems:-

The system that produces bounded output for bounded input is said to be stable system.

It is generally called as BIBO (Bounded Input Bounded Output) Stable system.

If the system gives unbounded output for bounded input is called unstable system.

For continuous time signal, the system that is absolutely integrable is called stable.

$$\int_{-\infty}^{\infty} h(t)\, dt < \infty$$

For DT system,

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$h(t)$, $h(n)$ → Impulse response of the system.

Example:-

check whether the following systems ~~with~~ stable or not.

(a) $y(t) = x(t) \sin 100\pi t$

$x(t)$ is bounded. The value of sine function is -1 to $+1$.

The sine function is multiplied by bounded input, therefore the output also bounded. The system is stable system.

(b) $t\, x(t)$

$x(t)$ is bounded but the 't' value varies from $-\infty$ to ∞. Therefore the output is unbounded. ∴ The system is unstable system.

(c) $y(n) = n x(n)$

→ In which the bounded input is multiplied by 'n'.

→ The 'n' value varies from $-\infty$ to ∞.

→ Therefore the output becomes unbounded.

→ ∴ The system is unstable.

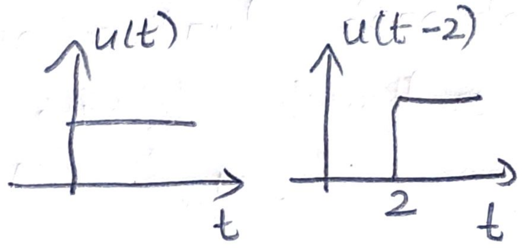
(d) $y(n) = a x(n)$

→ In which the bounded input is multiplied by constant value.

→ So the output is also bounded.

→ Therefore the system is stable.

e) $h(t) = e^{3t} u(t-2)$



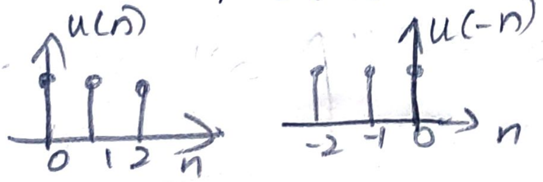
$u(t-2) = \begin{cases} 1 & \text{for } t \geq 2 \\ 0 & \text{otherwise.} \end{cases}$

$$\int_{-\infty}^{\infty} h(t)\,dt = \int_{2}^{\infty} e^{3t}\,dt = \frac{1}{2}\left[\frac{e^{3t}}{3}\right]_2^{\infty}$$

$$= \infty$$

∴ The system is unstable.

f) $h(n) = 4^n u(-n)$



$u(n) = \begin{cases} 1 & n \leq 0 \\ 0 & n > 0 \end{cases}$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{1} 4^n = \sum_{n=1}^{\infty} 4^{-n} = \sum_{n=1}^{\infty} \left(\tfrac{1}{4}\right)^n = \frac{1/4}{1-1/4}$$

$$= \frac{1/4}{3/4} = 1/3 < \infty$$

∴ The system is stable.