JEPPIAAR INSTITUTE OF TECHNOLOGY

“Self-Belief | Self Discipline | Self Respect”

DEPARTMENT

OF

ELECTRONICS & COMMUNICATION ENGINEERING

LECTURE NOTES

EC8553/ DISCRETE-TIME SIGNAL PROCESSING

(Regulation 2017)

Year/Semester: III/V ECE

2021 – 2022

Prepared by

Mrs.S.Mary Cynthia

Assistant Professor/ECE

EC8553 **DISCRETE-TIME SIGNAL PROCESSING** **L T P C**
4 0 0 4

OBJECTIVES:

- To learn discrete fourier transform, properties of DFT and its application to linear filtering
- To understand the characteristics of digital filters, design digital IIR and FIR filters and apply these filters to filter undesirable signals in various frequency bands
- To understand the effects of finite precision representation on digital filters
- To understand the fundamental concepts of multi rate signal processing and its applications
- To introduce the concepts of adaptive filters and its application to communication engineering.

UNIT I - DISCRETE FOURIER TRANSFORM **12**

Review of signals and systems, concept of frequency in discrete-time signals, summary of analysis & synthesis equations for FT & DTFT, frequency domain sampling, and Discrete Fourier transform (DFT) - deriving DFT from DTFT, properties of DFT - periodicity, symmetry, circular convolution. Linear filtering using DFT. Filtering long data sequences - overlap save and overlap add method. Fast computation of DFT - Radix-2 Decimation-in-time (DIT) Fast Fourier transform (FFT), Decimation-in-frequency (DIF) Fast Fourier transform (FFT). Linear filtering using FFT.

UNIT II - INFINITE IMPULSE RESPONSE FILTERS **12**

Characteristics of practical frequency selective filters. Characteristics of commonly used analog filters - Butterworth filters, Chebyshev filters. Design of IIR filters from analog filters (LPF, HPF, BPF, BRF) - Approximation of derivatives, Impulse invariance method, Bilinear transformation. Frequency transformation in the analog domain. Structure of IIR filter - direct form I, direct form II, Cascade, parallel realizations.

UNIT III - FINITE IMPULSE RESPONSE FILTERS **12**

Design of FIR filters - symmetric and Anti-symmetric FIR filters - design of linear phase FIR filters using Fourier series method - FIR filter design using windows (Rectangular, Hamming and Hanning window), Frequency sampling method. FIR filter structures - linear phase structure, direct form realizations.

UNIT IV FINITE WORD LENGTH EFFECTS **12**

Fixed point and floating point number representation - ADC - quantization - truncation and rounding - quantization noise - input / output quantization - coefficient quantization error - product quantization error - overflow error - limit cycle oscillations due to product quantization and summation - scaling to prevent overflow.

UNIT V INTRODUCTION TO DIGITAL SIGNAL PROCESSORS **12**

DSP functionalities - circular buffering – DSP architecture – Fixed and Floating point architecture principles – Programming – Application examples.

TOTAL: 60 PERIODS

OUTCOMES:

After studying this course, the student should be able to:

- Apply DFT for the analysis of digital signals and systems
- Design IIR and FIR filters
- Characterize the effects of finite precision representation on digital filters
- Design multirate filters
- Apply adaptive filters appropriately in communication systems.

TEXT BOOKS:

1. John G. Proakis & Dimitris G.Manolakis, ―Digital Signal Processing – Principles, Algorithms & Applications‖, Fourth Edition, Pearson Education / Prentice Hall, 2007. (UNIT I – V)

REFERENCES

1 Emmanuel C. Ifeachor & Barrie. W. Jervis, ―Digital Signal Processing‖, Second Edition, Pearson Education / Prentice Hall, 2002.
2. A. V. Oppenheim, R.W. Schafer and J.R. Buck, ―Discrete-Time Signal Processing‖, 8th Indian Reprint, Pearson, 2004.
3. Sanjit K. Mitra, ―Digital Signal Processing – A Computer Based Approach‖, Tata Mc Graw Hill, 2007.
4. Andreas Antoniou, ―Digital Signal Processing‖, Tata Mc Graw Hill, 2006.

EC 8553 Discrete - Time Signal processing.

Unit - I.

Discrete Fourier Transform

Syllabus:-

Review of signals and systems, Concept of frequency in discrete-time signals, Summary of analysis & synthesis equations for FT & DTFT, Frequency domain sampling, Discrete-Fourier-Transform (DFT) - deriving DFT from DTFT, Properties of DFT - periodicity, Symmetry, circular Convolution, Linear filtering using DFT - Filtering long data sequences - overlap save and overlap add method; Fast Computation of DFT, Radix-2 Decimation-in-time (DIT), Fast Fourier transform (FFT), Decimation-in-frequency (DIF) Linear filtering using FFT.

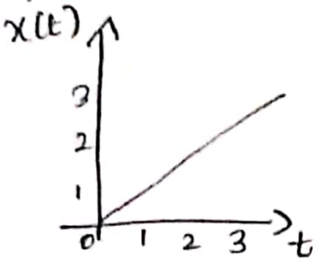
Review of Signals and Systems:-

Signal - A signal is defined as the physical quantity that varies with time, space or any other independent variables.

Continuous-time Signals (CT):

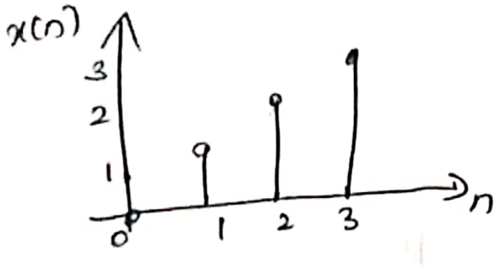
The signals that are defined for every instant of time are known as continuous-time signals. They are denoted by $x(t)$.

Example:-



Discrete-time signal (DT):-

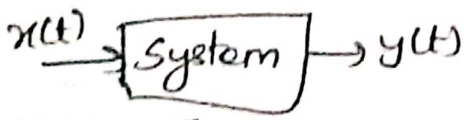
The signals that are defined at discrete-instant of time are known as discrete-time signals. They are continuous in amplitude and discrete-in-time. They are denoted by $x(n)$



System:-

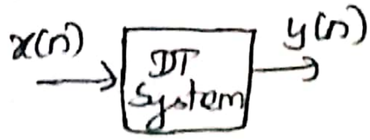
It is defined as a physical device that generates a response or output signal, for a given input signal.

CT



$y(t) = T[x(t)]$

DT



$y(n) = T[x(n)]$

$y(t)$ is the transformed form of $x(t)$.
$y(n)$ is the transformed form of $x(n)$.

Discrete Fourier Transform:-

It is defined as.

$$x(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

Where $N \rightarrow$ length of sequence

$n = 0, 1, \ldots N-1$

$k = 0, 1, \ldots N-1$

EX. 1.1. Compute DFT $x(n) = \{1, 0, 1, 0\}$

$x(n) = \{1, 0, 1, 0\}$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$N = 4,$

$n = 0, 1, 2, \ldots N-1 \Rightarrow n = 0, 1, 2, 3.$

$k = 0, 1, 2, \ldots N-1 \Rightarrow k = 0, 1, 2, 3$

$$X(k) = \sum_{n=0}^{4-1} x(n)\, e^{-j\frac{\cancel{2}\pi nk}{\cancel{4}2}}$$

$k = 0, 1, \ldots N-1$

$\boxed{k = 0, 1, 2, 3}$

$$X(K) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi nk}{2}}$$

$$x(K) = x(0) e^{-j\frac{\pi \times 0 \times k}{2}} + x(1) e^{-j\frac{\pi k}{2}} + x(2) e^{-j\frac{\pi 2k}{2}} + x(3) e^{-j\frac{3\pi k}{2}}$$

Given,

$x(0) = 1$

$x(1) = 0$

$x(2) = 1$

$x(3) = 0$

$$x(k) = 1 \times 1 + 0 + 1 . e^{-j\frac{\pi \cancel{2}k}{\cancel{2}}} + 0$$

$$\boxed{X(k) = 1 + e^{-j\pi k}}$$

$k = 0,$ $X(0) = 1 + e^{-j\pi 0} = 1 + 1 \Rightarrow \boxed{X(0) = 2}$

$k = 1,$ $x(1) = 1 + e^{-j\pi \times 1} \Rightarrow X(1) = 1 - 1 \Rightarrow \boxed{X(1) = 0}$

$= 1 + \cos\pi - j\sin\pi$

$k=2,\quad x(2) = 1 + e^{-j2\pi}$

$x(2) = 1 + (\cos 2\pi - j\sin 2\pi) = 1+1-0 \Rightarrow \boxed{x(2) = 2}$

$k=3,\quad x(3) = 1 + e^{-j3\pi}$

$= 1 + (\cos 3\pi - j\sin 3\pi)$

$= 1 + (-1 - 0)$

$\boxed{x(3) = 0}$

ie) $X(K) = \{2, 0, 2, 0\}$

Ex.1.2. compute four sequence of DFT $x(n) = \{0, 1, 2, 3\}$

$$x(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$N=4 \Rightarrow n = 0, 1, 2, 3$ & $k = 0, 1, 2, 3$

$$x(K) = \sum_{n=0}^{4-1} x(n) e^{-j\frac{2\pi nk}{4}}$$

$$x(K) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi nk}{2}}$$

$$x(K) = x(0)\times 1 + x(1) e^{-j\frac{\pi k}{2}} + x(2) e^{-j\frac{2\pi k}{2}} + x(3) e^{-j\frac{3\pi k}{3}}$$

$x(0) = 0,\quad x(1) = 1,\quad x(2) = 2,\quad x(3) = 3.$

$x(K) = 0 + 1.e^{-j\pi k/2} + 2.e^{-j\pi k} + 3e^{-j3\pi k/2}$

$$\boxed{x(K) = e^{-j\pi k/2} + 2e^{-j\pi k} + 3e^{-j3\pi k/2}}$$

$k=0,\quad x(0) = 1 + 2 + 3$

$\boxed{x(0) = 6}$

$k=1$, $x(1) = e^{-j\pi/2} + 2e^{-j\pi(1)} + 3e^{-j3\pi/2}$

$= -j + 2(\cos\pi - j\sin\pi) + 3(\cos 3\pi/2 - j\sin 3\pi/2)$

$= -j + 2(-1) + 3j$

$= -j - 2 + 3j$

$\boxed{x(1) = -2+2j}$

$k=2$, $x(2) = e^{-j\pi} + 2e^{-j2\pi} + 3e^{-j3\pi}$

$= -1 + 2(1) + 3(-1)$

$= -1 + 2 - 3$

$\boxed{x(2) = -2}$

$k=3$, $x(3) = e^{-j3\pi/2} + 2e^{-j\pi 3} + 3e^{-j\pi 9/2}$

$= -j + 2(-1) + 3(-j)$

$\boxed{x(3) = -2 - 2j}$

$x(k) = \{6, -2+2j, -2, -2-2j\}$

EX.1.3. Circular convolution of two finite duration sequences $x(n) = \{1, -1, -2, 3, -1\}$ and $h(n) = \{1, 2, 3\}$

Given :-

$x(n) = \{1, -1, 2, 3, -1\}$

$h(n) = \{1, 2, 3\}$

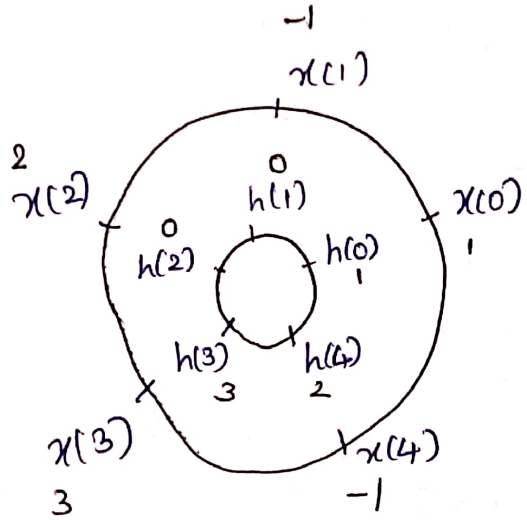
$y(n) = x(n) * h(n)$

Step 1:- (y(0))

Outer circle → Anti clockwise

Inner circle → Clockwise.

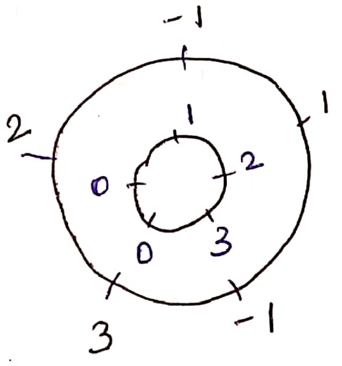
changing in anticlockwise



$y(0) = 1+0+0+9-2 = 8$

$\boxed{y(0) = 8}$

Step 2:- y(1)

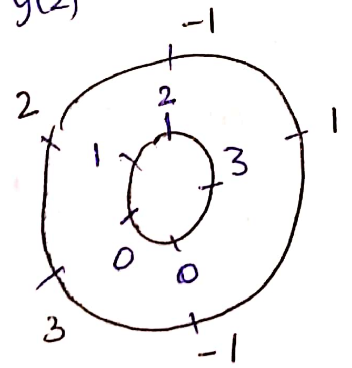


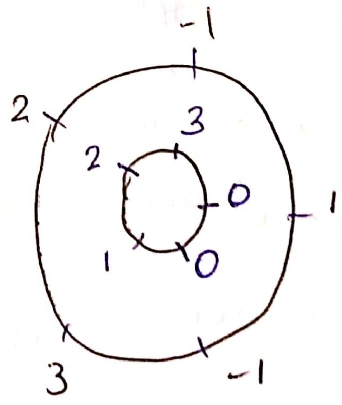
$y(1) = 0+0-3+2-1$

$\boxed{y(1) = -2}$

Step 4: y(3)

Step 3:- y(2)





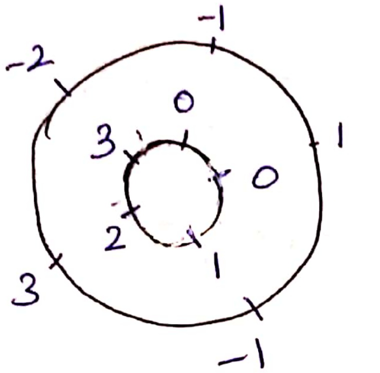
$y(2) = 2-2+3$

$y(2) = 3$

$y(3) = 3+4-3$

$y(3) = 4$

step 5: $y(4)$



$$y(4) = -1+6-6$$
$$y(4) = -1$$

$$y(n) = \{8, -2, 3, 4, -1\}$$

Inverse Discrete Fourier Transform (IDFT)

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(K)\, e^{-j\frac{2\pi nk}{N}}$$

Ex. 1.4. perform Circular Convolution $x(n) = \{1, 1, 2, 1\}$

$h(n) = \{1, 2, 3, 4\}$ using DFT & IDFT.

$$y(n) = x(n) * h(n) \quad \& \quad Y(K) = X(K)\, H(K)$$

To find $x(K)$,

$$x(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

Where $N=4$, $k = 0, 1, 2, 3$
$n = 0, 1, 2, 3$

$$x(K) = \sum_{n=0}^{4-1} x(n)\, e^{-j\cancel{2}\pi nk/\cancel{4}2}$$

$$x(K) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi nk}{2}}$$

$$x(K) = x(0)\times 1 + x(1)\, e^{-j\pi k/2} + x(2)\, e^{-j\frac{\pi 2k}{2}} + x(3)\, e^{-j\frac{\pi 3k}{2}}$$

$x(0) = 1$, $x(1) = 1$, $x(2) = 2$, $x(3) = 1$

$$x(K) = 1 + e^{-j\pi k/2} + 2e^{-j\pi k} + e^{-j\pi 3k/2} \quad \text{—(1)}$$

$K=0,$

$$x(0)=1+1+2+1$$

$$\boxed{X(0)=5}$$

$k=1,$

$$x(1)=1+e^{-j\pi/2}+2e^{-j\pi}+e^{j3\pi/2}$$

$$=1+(-j)+2(-1)+(-j)$$

$$=1-j-2+j$$

$$\boxed{x(1)=-1}$$

$k=2,$

$$x(2)=1+e^{-j\pi}+2e^{-j2\pi}+e^{-j\pi 6/2}$$

$$=1+(-1)+2(1)+\cos\pi(6/2)-j\sin\pi(6/2)$$

$$=1-1+2-1$$

$$\boxed{x(2)=1}$$

$k=3,$

$$x(3)=1+e^{-j\pi 3/2}+2e^{-j3\pi}+e^{-j\pi 9/2}$$

$$=1j+2(-1)+(-j)$$

$$=1-j-2+j$$

$$\boxed{x(3)=-1}$$

$$x(K)=\{5,-1,1,-1\}$$

$h(n)=\{1,2,3,4\}$

$$H(K)=\sum_{n=0}^{N-1}h(n)\,e^{-j2\pi nk/N}$$

$$H(K)=\sum_{n=0}^{4-1}h(n)\,e^{-j2\pi nk/4}$$

$$H(k) = \sum_{n=0}^{3} h(n) e^{-j\pi n k/2}$$

$$H(k) = h(0)\times 1 + h(1) e^{-j\pi k/2} + h(2) e^{-j2k/2} + e^{-j3\pi k/2} h(3)$$

$h(0) = 1$, $h(1) = 2$, $h(2) = 3$, $h(3) = 4$.

$$H(k) = 1 + 2e^{-j\pi k/2} + 3e^{-j\pi k} + 4e^{-j3\pi k/2}$$

$$H(k) = 1 + 2e^{-jk\pi/2} + 3e^{-j\pi k} + 4e^{-j3\pi k/2} \quad \text{——— (2)}$$

Where $k = 0, 1, 2, 3$.

$k=0$, $h(0) = 1+2+3+4$

$$\boxed{h(0) = 10}$$

$k=1$, $h(1) = 1 + 2e^{-j\pi/2} + 3e^{-j\pi} + 4e^{-j\pi 3/2}$

$= 1 + 2(-j) + 3(-1) + 4(j)$

$= 1 - 2j - 3 + 4j$

$$\boxed{h(1) = -2 + 2j}$$

$k=2$, $h(2) = 1 + 2e^{-j\pi} + 3e^{-j2\pi} + 4e^{-j3\pi}$

$= 1 + 2(-1) + 3(1) + 4(-1)$

$= 1 - 2 + 3 - 4$

$$\boxed{h(2) = -2}$$

$k=3$, $h(3) = 1 + 2e^{-j\pi 3/2} + 3e^{-j\pi 3} + 4e^{-j\pi 9/2}$

$= 1 + 2j + 3(-1) - 4j$

$$\boxed{h(3) = -2 - 2j}$$

$$x(K) = \{5, -1, 1, -1\}$$

$$h(K) = \{10, -2+2j, -2, -2-2j\}$$

$$y(K) = x(K) * h(K)$$

$$y(K) = \{5, -1, 1, -1\} * \{10, -2+2j, -2, -2-2j\}$$

DFT sequence is,

$$y(K) = \{50, 2-2j, -2, 2+2j\}$$

$y(n)$ is calculated using IDFT,

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(K) e^{\frac{j2\pi nk}{N}}$$

$$y(n) = \frac{1}{4} \sum_{k=0}^{3} y(k) e^{\frac{j2\pi nk}{\cancel{4}\,2}}$$

$$y(n) = \frac{1}{4} \sum_{k=0}^{3} y(k) e^{\frac{j\pi nk}{2}}$$

$$y(n) = \frac{1}{4} \{y(0) + y(1) e^{j\pi n/2} + y(2) e^{j\cancel{2}\pi n/\cancel{2}} + y(3) e^{j3\pi n/2}\}$$

$$y(n) = \frac{1}{4} \{50 + (2-2j) e^{j\pi n/2} - 2e^{j\pi n} + (2+2j) e^{j3\pi n/2}\} \quad \text{—(1)}$$

where $n = 0, 1, 2, 3$.

$n=0$,

$$y(0) = \frac{1}{4} \{50 + 2 - 2j - 2 + 2 + 2j\}$$

$$\boxed{y(0) = 13}$$

$n=1$,

$$y(1) = \frac{1}{4} \{50 + (2-2j) e^{j\pi/2} - 2e^{j\pi} + (2+2j) e^{j3\pi/2}\}$$

$$= \frac{1}{4}\{50 + (2-2j)j - 2(-1) + (2+2j)(-j)\}$$

$$= \frac{1}{4}\{56\}$$

$$\boxed{y(1) = 14}$$

$n=2$, $$y(2) = \frac{1}{4}\{50 + (2-2j)e^{j\pi} - 2e^{j\pi 2} + (2+2j)e^{j3\pi}\}$$

$$= \frac{1}{4}\{50 + (2-2j)(-1) - 2(1) + (2+2j)(-1)\}$$

$$= \frac{1}{4}\{44\}$$

$$\boxed{y(2) = 11}$$

$$y(3) = \frac{1}{4}\{50 + (2-2j)e^{j3\pi/2} - 2e^{j3\pi} + (2+2j)e^{j9\pi/2}\}$$

$$= \frac{1}{4}\{50 + (2-2j)(-j) - 2(-1) + (2+2j)(j)\}$$

$$= \frac{1}{4}\{48\}$$

$$\boxed{y(3) = 12}$$

The Output Sequence is,

$$y(n) = \{13, 14, 11, 12\}$$

Convolution:-

Formula:-

$$y(n) = x(n) * h(n)$$

where $y(n)$ - Output, $x(n)$ - Input, $h(n)$ - Impulse Response

In frequency domain,

$$Y(K) = X(K)H(K)$$

Linear Convolution:

Ex.1.5. Determine Output of the system $x(n) = \{1, -1, 2, -2\}$ and $h(n) = \{4, 3, 2, 1\}$ using linear/ tabular Convolution.

$$y(n) = x(n) * h(n)$$

$x(n) \backslash h(n)$	4	3	2	1
1	4	3	2	1
-1	-4	-3	-2	-1
2	8	6	4	2
-2	-8	-6	-4	-2

$$y(n) = \{4, -1, 7, -3, -3, -2, -2\}$$

Ex.1.6. Using linear Convolution find $x(n) = x_1(n) * x_2(n)$ and $x_1(n) = \{1, 2, 3, 4\}$ & $x_2(n) = \{\sin \frac{n\pi}{4}\}$ if $N = 4$.

$n = 0, 1, \ldots N-1$

$\Rightarrow n = 0, 1, 2, 3$

$x_2(0) = \sin 0 = 0$

$x_2(1) = \sin \pi/4 = 0.707$

$x_2(2) = \sin 2\pi/4 = 1$

$x_2(3) = \sin 3\pi/4 = 0.707$

(ie) $x_2(n) = \{0, 0.707, 1, 0.707\}$

$x_1(n)$ \ $x_2(n)$	0	0.707	1	0.707
1	0	0.707	1	0.707
2	0	1.414	2	1.414
3	0	2.121	3	2.121
4	0	2.828	4	2.828

$$x(n) = \{0,\ 0.707,\ 2.414,\ 4.828,\ 7.242,\ 6.121,\ 2.828\}$$

EX.1.7. Using matrix convolution find $x(n) = x_1(n) * x_2(n)$

$x_1(n) = \{1, 1, -1, 0\}$ & $x_2(n) = \{1, 3, 5, 7\}$

$$x(n) = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-5+7 \\ 1+3+0-7 \\ -1+3+5+0 \\ 0-3+5+7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -3 \\ 7 \\ 9 \end{bmatrix}$$

$$x(n) = \{3, -3, 7, 9\}$$

Cross correlation :-

It is denoted by $r_{xl} = x_1(l) * x_2(-l)$

EX.1.8. Find cross correlation of the given signals $x_1(l) = \{1, 1, -1, 0\}$ and $x_2(l) = \{1, 3, 5, 7\}$

Given :- $x_1(l) = \{1, 1, -1, 0\}$

$x_2(l) = \{1, 3, 5, 7\}$

$x_2(-l) = \{7, 5, 3, 1\}$

$x_1(l)$ \ $x_2(-l)$	7	5	3	1
1	7	5	3	1
1	7	5	3	1
-1	-7	-5	-3	-1
0	0	0	0	0

$r_{x_1x_2} = \{7, 12, 1, -1, -2, -1, 0\}$

Linear Filtering (or) Filtering methods of DFT.

1. Overlap Add method → using linear convolution
2. Overlap Save method → using matrix / Circular convolution.

→ These methods used for low impulse signal to process high or long duration input sequence.

EX. 1.9. perform linear convolution of finite duration sequence $h(n) = \{1, 2\}$. I/P sequence $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$.

Given :- $h(n) = \{1, 2\}$

$x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

Overlap Add method :-

using linear Convolution.

$x_1(n) = \{1, 2\}$; $x_2(n) = \{-1, 2\}$, $x_3(n) = \{3, -2\}$

$x_4(n) = \{-3, -1\}$, $x_5(n) = \{1, 1\}$,. $x_6(n) = \{2, -1\}$

step 1 : $h(n) = \{1, 2\}$

$y_1(n) = x_1(n) * h(n)$

$y_1(n) = \{1, 2\} * \{1, 2\}$

$x_1(n)$ \ $h(n)$	1	2
1	1	2
2	2	4

$y_1(n) = \{1, 4, 4\}$

step 2 :-

$y_2(n) = \{-1, 2\} * \{1, 2\}$

$x_2(n)$ \ $h(n)$	1	2
-1	-1	-2
2	2	4

$y_2(n) = \{-1, 0, 4\}$

step 3 :-

$y_3(n) = \{3, -2\} * \{1, 2\}$

$x_3(n)$ \ $h(n)$	1	2
3	3	6
-2	-2	-4

$y_3(n) = \{3, 4, -4\}$

step 4 :-

$y_4(n) = \{1, 2\} * \{-3, -1\}$

$x_4(n)$ \ $h(n)$	1	2
-3	-3	-6
-1	-1	-2

$y_4(n) = \{-3, -7, -2\}$

step 5 :-

$$y_5(n) = \{1, 2\} * \{1, 1\}$$

$x_5(n) \backslash h(n)$	1	2
1	1	2
1	1	2

$$y_5(n) = \{1, 3, 2\}$$

step 6 :-

$$y_6(n) = \{2, -1\} * \{1, 2\}$$

$x_6(n) \backslash h(n)$	1	2
2	2	4
-1	-1	-2

$$y_6(n) = \{2, 3, -2\}$$

Tabulation :-

n	0	1	2	3	4	5	6	7	8	9	10	11	12
$y_1(n)$	1	4	4										
$y_2(n)$			-1	0	4								
$y_3(n)$					3	4	-4						
$y_4(n)$							-3	-7	-2				
$y_5(n)$									1	3	2		
$y_6(n)$											2	3	-2
	1	4	3	0	7	4	-7	-7	-1	3	4	3	-2

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Overlap- Save Method :-

Using Circular convolution,

$(N \times M)-1$ & "M is always 2".

N – h(n) length.

$x(n)=\{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

$x_1(n) = \{1, 2, -1\}$ $x_4(n) = \{-3, -1, 1\}$

$x_2(n) = \{-1, 2, 3\}$ $x_5(n) = \{1, 1, 2\}$

$x_3(n) = \{3, -2, -3\}$ $x_6(n) = \{2, -1, 0\}$

Step 1: $y_1(n) = x_1(n) * h(n)$

$$y_1(n) = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$y_1(n) = \begin{bmatrix} 1-2+0 \\ 2+2+0 \\ -1+4+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$$

$y_1(n) = \{-1, 4, 3\}$

step 2:- $y_2(n) = x_2(n) * h(n)$

$$y_2(n) = \begin{bmatrix} -1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+6+0 \\ 2-2+0 \\ 3+4+0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 7 \end{bmatrix}$$

$y_2(n) = [5 \quad 0 \quad 7]$

step 3:- $y_3(n) = x_3(n) * h(n)$

$$y_3(n) = \begin{bmatrix} 3 & -3 & -2 \\ -2 & 3 & -3 \\ -3 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3-6 \\ -2+6 \\ -3-4 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ -7 \end{bmatrix}$$

$$y_3(n) = [-3, 4, 7]$$

Step 4:-

$$y_4(n) = x_4(n) * h(n)$$

$$y_4(n) = \begin{bmatrix} -3 & 1 & -1 \\ -1 & -3 & 1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -3+2 \\ -1-6 \\ 1-2 \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ -1 \end{bmatrix}$$

$$y_4(n) = [-1 \quad -7 \quad -1]$$

Step 5:-

$$y_5(n) = x_5(n) * h(n)$$

$$y_5(n) = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+4 \\ 1+2 \\ 2+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$$

$$y_5(n) = [5 \; 3 \; 4]$$

Step 6:

$$y_6(n) = x_6(n) * h(n)$$

$$y_6(n) = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1+4 \\ 0-2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

$$y_6(n) = [2 \quad 3 \quad -2]$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12
$y_1(n)$	-1	4	3										
$y_2(n)$			~~5~~	0	7								
$y_3(n)$					~~-3~~	4	-7						
$y_4(n)$							~~1~~	-7	-1				
$y_5(n)$									~~5~~	3	4		
$y_6(n)$											~~2~~	3	-2
	-1	4	3	0	7	4	-7	-7	-1	3	4	3	-2

* 4 3 0 7 4 -7 -7 -1 3 4 3 -2

$y(n) = \{*, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$

Properties of DFT Transform :-

1. Linearity
2. Time shifting
3. Time Reversal
4. Parseval's theorem (or) Energy density (or) Power Spectrum

1) Linearity :-

By superposition principle,

Weighted sum of Input is equal to the weighted sum of output is called linearity.

If DFT of $x(n) = X(K)$, DFT of $x_1(n) = X_1(K)$

& DFT of $x_2(n) = X_2(K)$

then $DFT[a x_1(n) + b x_2(n)] = a X_1(K) + b X_2(K)$

Where a, b are arbitary constants.

Proof :-

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n) e^{-j\frac{2\pi nk}{N}}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi nk/N}$$

$$\text{L.H.S} = DFT[a\, x_1(n) + b x_2(n)] = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} [a x_1(n) + b x_2(n)] e^{-j2\pi nk/N}$$

$$= \sum_{n=0}^{N-1} a x_1(n) e^{-j2\pi nk/N} + \sum_{n=0}^{N-1} b x_2(n) e^{-j\frac{2\pi nk}{N}}$$

$$DFT[a x_1(n) + b x_2(n)] = a X_1(K) + b X_2(K)$$

2) Time shifting :-

If $DFT[x(n)] = X(K)$

then $DFT[x(n-m)] = X(k) e^{-j\frac{2\pi mk}{N}}$

& $DFT[x(n+m)] = X(K) e^{j2\pi mk/N}$

Proof :-

$$DFT[x(n)] = X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$DFT[x(n-m)] = \sum_{n=0}^{N-1} x(n-m) e^{-j2\pi nk/N}$$

Assume $P = n-m \Rightarrow n = p+m$

when $n=0 \Rightarrow p=0$, $DFT[x(n-m)] = \sum_{p=0}^{N-1} x(p) e^{-j2\pi(p+m)k/N}$

$$DFT[x(n-m)] = \sum_{p=0}^{N-1} x(p) e^{-j2\pi pk/N} e^{-j2\pi mk/N}$$

$$DF[x(n-m)] = X(K) e^{-j2\pi mk/N}$$

3) Time Reversal :-

If DFT of $x(n) = X(k)$

then DFT of $x(N-n) = X(N-k)$

Proof :-

$$\text{DFT of } x(n) = X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$\text{DFT of } x(N-n) = \sum_{n=0}^{N-1} x(N-n) e^{-j2\pi nk/N}$$

$$m = N-n \Rightarrow n = N-m$$

$$\text{DFT}[x(N-n)] = \sum_{m=0}^{N-1} x(m) e^{-j2\pi (N-m)k/N}$$

$$\text{DFT}[x(N-n)] = \sum_{m=0}^{N-1} x(m) e^{-j2\pi \cancel{N}k/\cancel{N}} e^{+j2\pi mk/N}$$

$$\boxed{\text{for } k = 0, 1, 2, \ldots \quad e^{j2\pi k} = 1}$$

$$\text{DFT}[x(N-n)] = X(N-k)$$

4) Parseval's Theorem :-

If $\text{DFT}[x(n)] = X(k)$

and $\text{DFT}[y(n)] = Y(k)$

then $$\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) Y^*(k)$$

Proof :-

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}, \qquad y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) e^{j2\pi nk/N}$$

$$\text{R.H.S} = \frac{1}{N} \sum_{k=0}^{N-1} X(k) Y^*(k) = \frac{1}{N} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \right] Y^*(k)$$

$$= \sum_{n=0}^{N-1} x(n) \left[\frac{1}{N} \sum_{k=0}^{N-1} Y^*(k) e^{-j2\pi nk/N} \right]$$

$$= \sum_{n=0}^{N-1} x(n) \left[\frac{1}{N} \sum_{k=0}^{N-1} Y(k) e^{j2\pi nk/N} \right]^* = \sum_{n=0}^{N-1} x(n) y^*(n)$$

FFT (Fast Fourier Transform):-

* It is an algorithm used to compute the DFT.

* It reduces the computation time required to compute discrete fourier transform.

Types of FFT:-

1. Decimation-in-time (DIT)
2. Decimation-in-frequency (DIF)

* For DIT the input is bit reversal while the output is in natural order, whereas for DIF, the input is in natural order while the output is bit reversal.

1. Decimation-in-time (DIT)

Bit reversal.

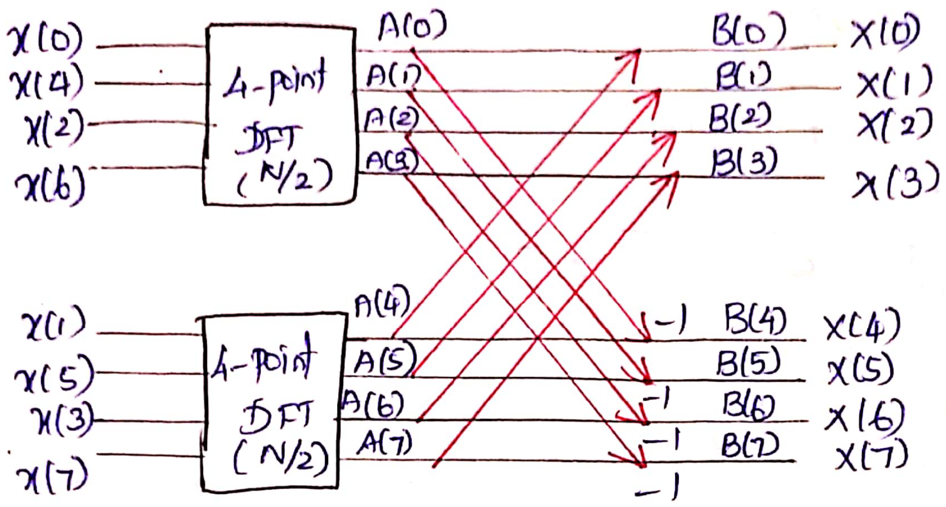
Decimal	Binary	Bit-reversal Order	Bit-reversal index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

even

→odd

Step 1 :

The DFT properties are divided to 4-points.



Output :-

$B(0) = A(0) + A(4)$

$B(1) = A(1) + A(5)$

$B(2) = A(2) + A(6)$

$B(3) = A(3) + A(7)$

$B(4) = A(0) - A(4)$

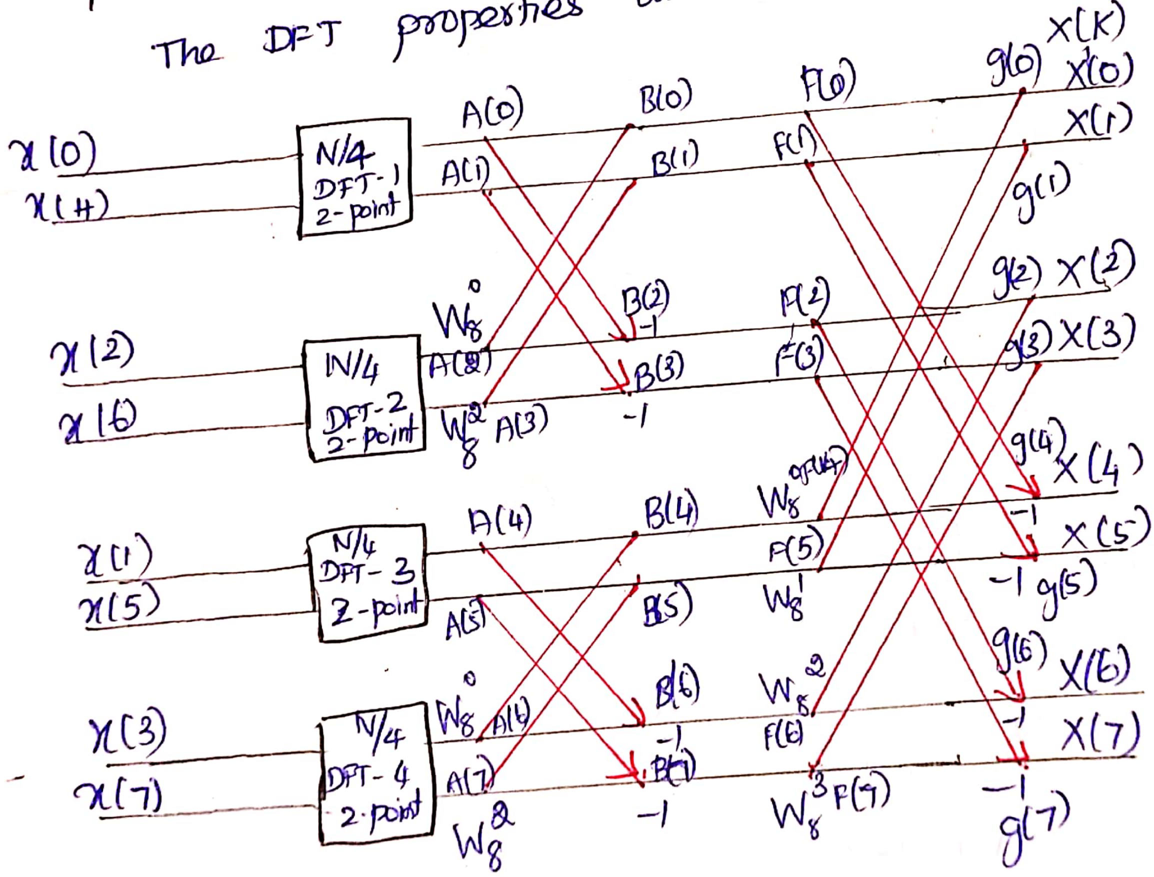
$B(5) = A(1) - A(5)$

$B(6) = A(2) - A(6)$

$B(7) = A(3) - A(7)$

Step 2 :-

The DFT properties are divided into 2-points.



Step 3 :-

Twiddle factor

$$W_N^k = e^{-j2\pi K/N}$$

N = 8 , when K = 0, $W_8^0 = e^0 = 1$

$$\boxed{W_8^0 = 1}$$

$k=1,$

$$W_8^1 = e^{-j\frac{2\pi}{8}} = e^{-j\pi/4}$$

$$= \cos\pi/4 - j\sin\pi/4$$

$$\boxed{W_8^1 = 0.707 - j0.707}$$

$k=2,$ $$W_8^2 = e^{-j\frac{2\times 2\pi}{8}} = e^{-j\pi/2}$$

$$\boxed{W_8^2 = -j}$$

$k=3,$ $$W_8^3 = e^{-j\frac{2\pi\times 3}{8}}$$

$$= e^{-j3\pi/4}$$

$$\boxed{W_8^3 = -0.707 - j0.707}$$

Step 4:-

The definition of DFT is,

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

For even values,

$$x(n) = x(2n)$$

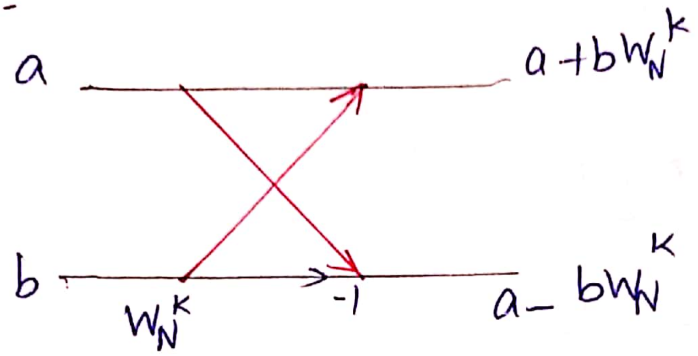
$$X(k) = \sum_{n=\text{even}}^{N-1} x(2n) e^{-j2\pi nk/N}$$

For odd values,

$$x(n) = x(2n+1)$$

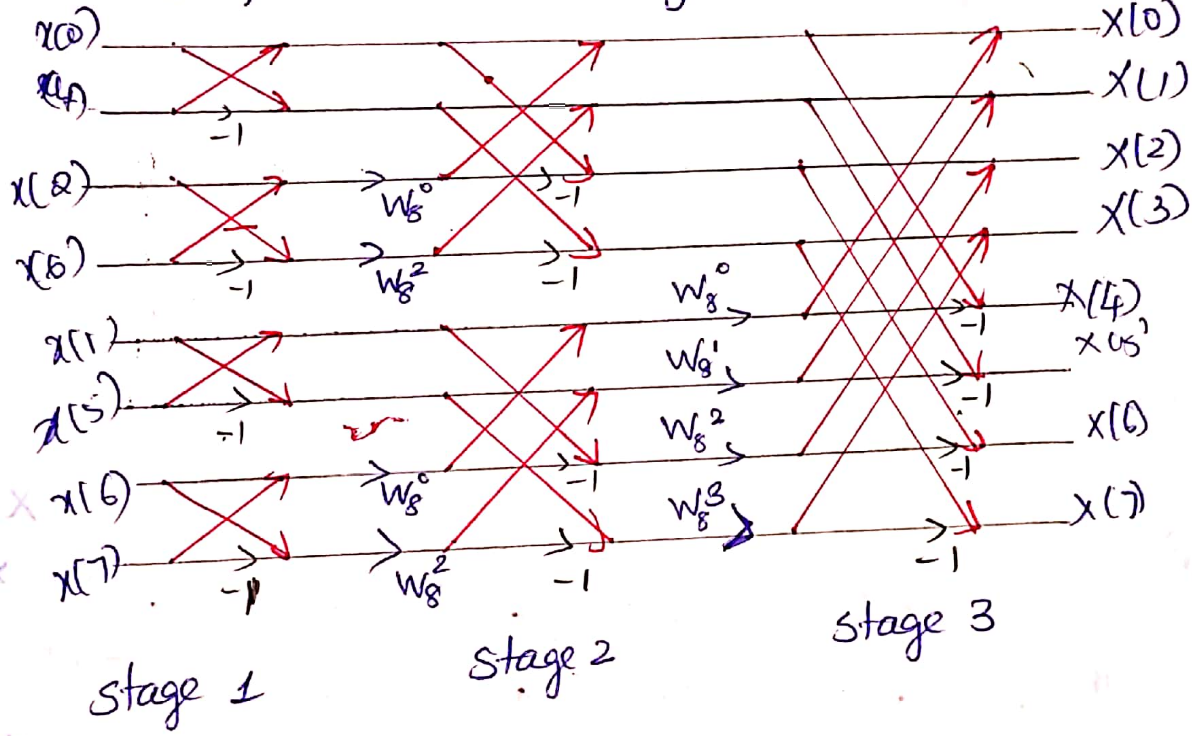
$$X(k) = \sum_{n=\text{odd}}^{N-1} x(2n+1) e^{-j2\pi nk/N}$$

step 5:-



step 6:-

Complete Butterfly structure for DIT is,



EX.1.10. Compute DFT of a sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIT-FFT algorithm.

Step 1: Twiddle factor:-

$$W_N^k = e^{-j2\pi k/N}$$

N = 8, when k = 0, $W_8^0 = e^0 = 1$

$$\boxed{W_8^0 = 1}$$

when $k=1$,

$$W_8^1 = e^{-j2\pi/8}$$

$$\boxed{W_8^1 = 0.707 - j0.707}$$

When $k=2$,

$$W_8^2 = e^{-j\frac{2\pi(2)}{8}}$$

$$\boxed{W_8^2 = -j}$$

When $k=3$,

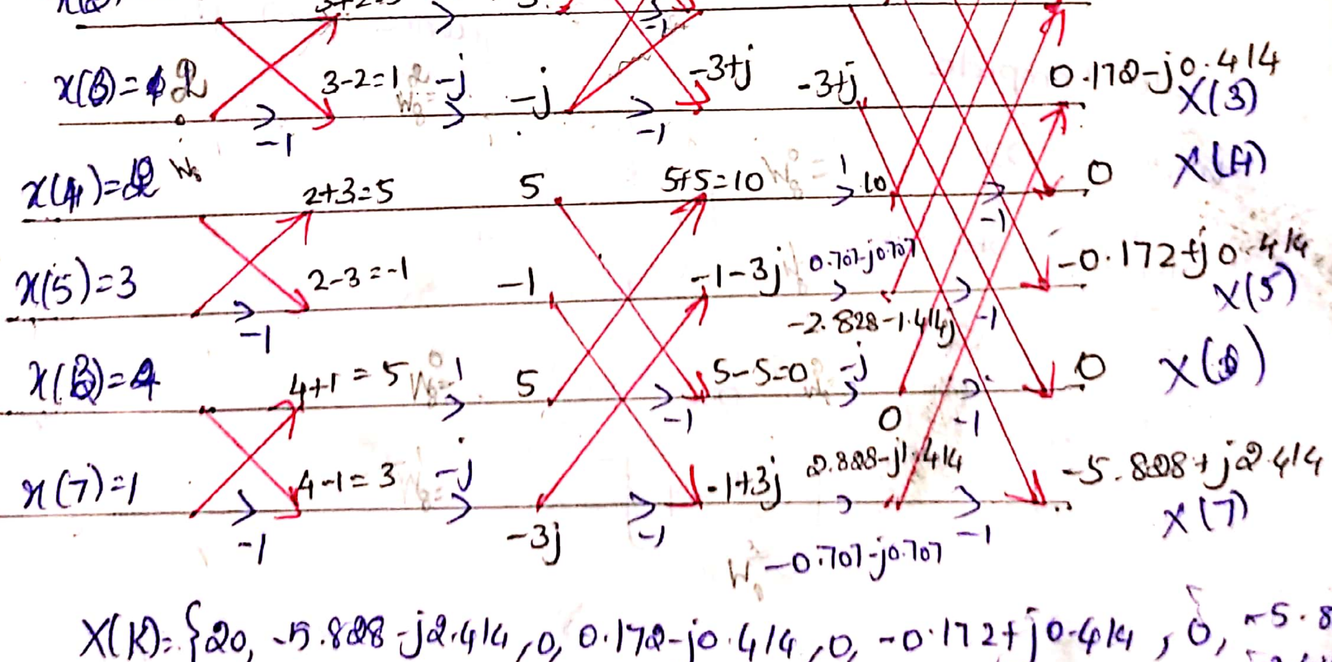
$$W_8^3 = e^{-j\frac{2\pi(3)}{8}}$$

$$W_8^3 = -0.707 - j0.707.$$

I/P sequence:-

$x(0)=1$, $x(1)=2$, $x(2)=3$, $x(3)=4$, $x(4)=4$

$x(5)=3$, $x(6)=2$, $x(7)=1$



$X(K) = \{20, -5.828 - j2.414, 0, 0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}$

Ex.1.11 Determine 4-point DFT of a sequence $x(n) = \{1, 2, 3, 4\}$ using DIT FFT algorithm.

$$x(n) = \{1, 2, 3, 4\}$$

Twiddle factor,

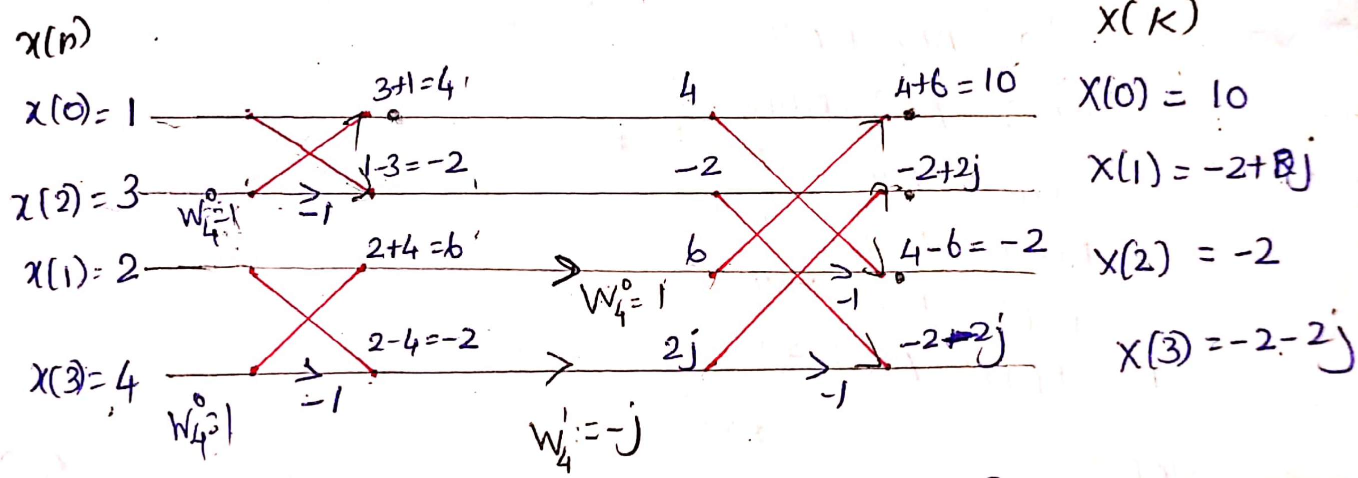
$$W_N^k = e^{-j2\pi k/N}$$

$N = 4$,

$k = 0$, $W_4^0 = e^0 = 1 \Rightarrow \boxed{W_4^0 = 1}$

$k = 1$, $W_4^1 = e^{-j\frac{2\pi \times 1}{4}} = e^{-j\pi/2} = -j$ $\quad \cos\pi/2 - j\sin\pi/2$

$$\boxed{W_4^1 = -j}$$

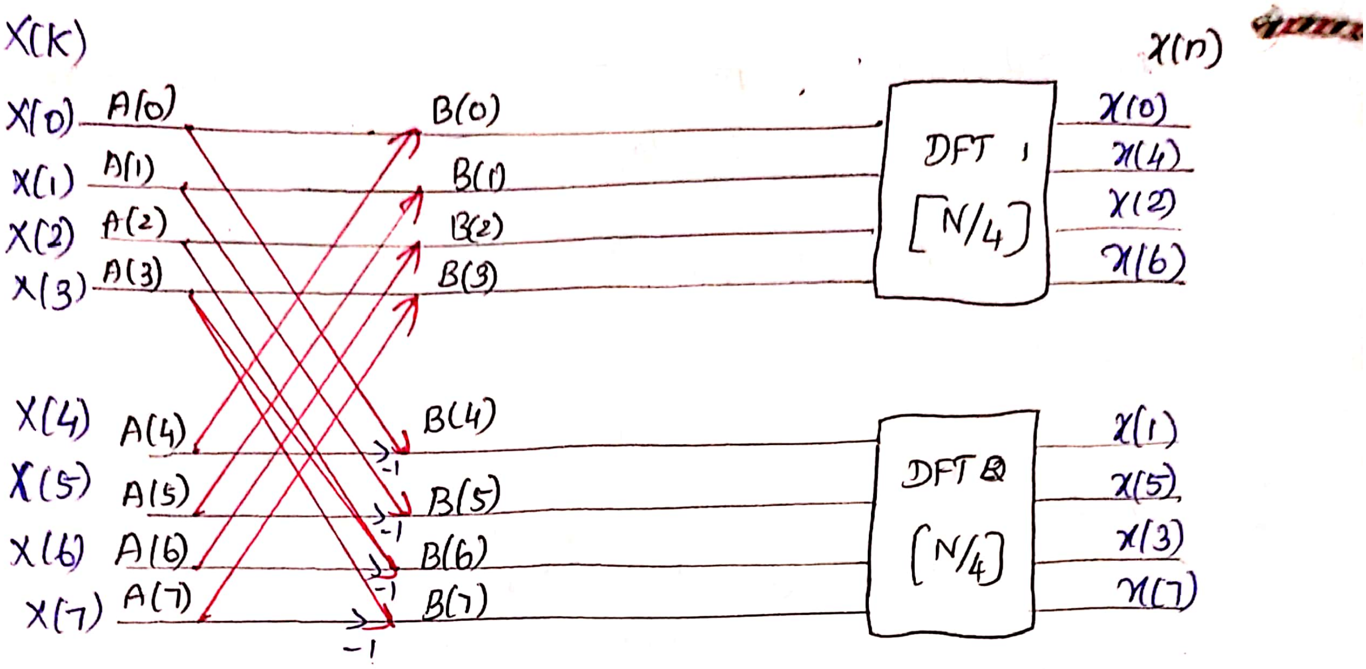


$$X(K) = \{10, -2+2j, -2, -2-2j\}$$

DIF (Decimation in Frequency)

Step 1: Divide DFT by N/4

N = 8



Output :-

$B(0) = A(0) + A(4)$

$B(1) = A(5) + A(1)$

$B(2) = A(2) + A(6)$

$B(3) = A(3) + A(7)$

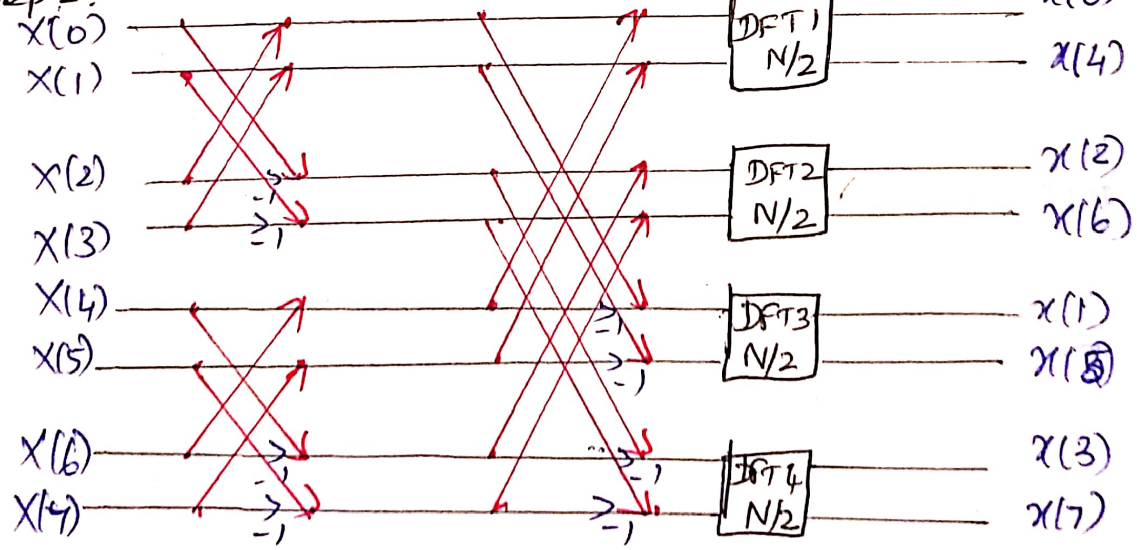
$B(4) = A(0) - A(4)$

$B(5) = A(1) - A(5)$

$B(6) = A(2) - A(6)$

$B(7) = A(3) - A(7)$

step 2:-



DFT of $x(n)$ is,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi k/N}$$

for odd values, $x(n) = x(2n+1)$

$$X(k) = \sum_{n=0}^{N-1} x(2n+1)\, e^{-j2\pi k/N}$$

for even values $x(n) = x(2n)$

$$X(k) = \sum_{n=0}^{N-1} x(2n)\, e^{-j2\pi k/N}$$

Twiddle factor.

$N=8$, $W_N^k = e^{-j2\pi k/N}$

$k=0$, $W_8^0 = e^0 = 1$

$$W_8^0 = 1$$

$k=1$, $W_8^1 = e^{-j2\pi/8} = e^{-j\pi/4}$

$$W_8^1 = 0.707 - j0.707$$

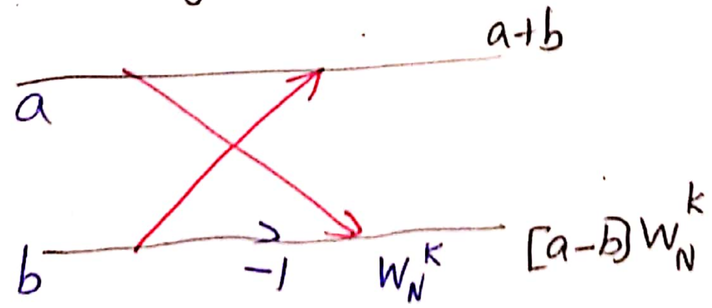
$k=2$, $W_8^2 = e^{-j\frac{2\pi\times 2}{8}} = e^{-j\pi/2}$

$$W_8^2 = -j$$

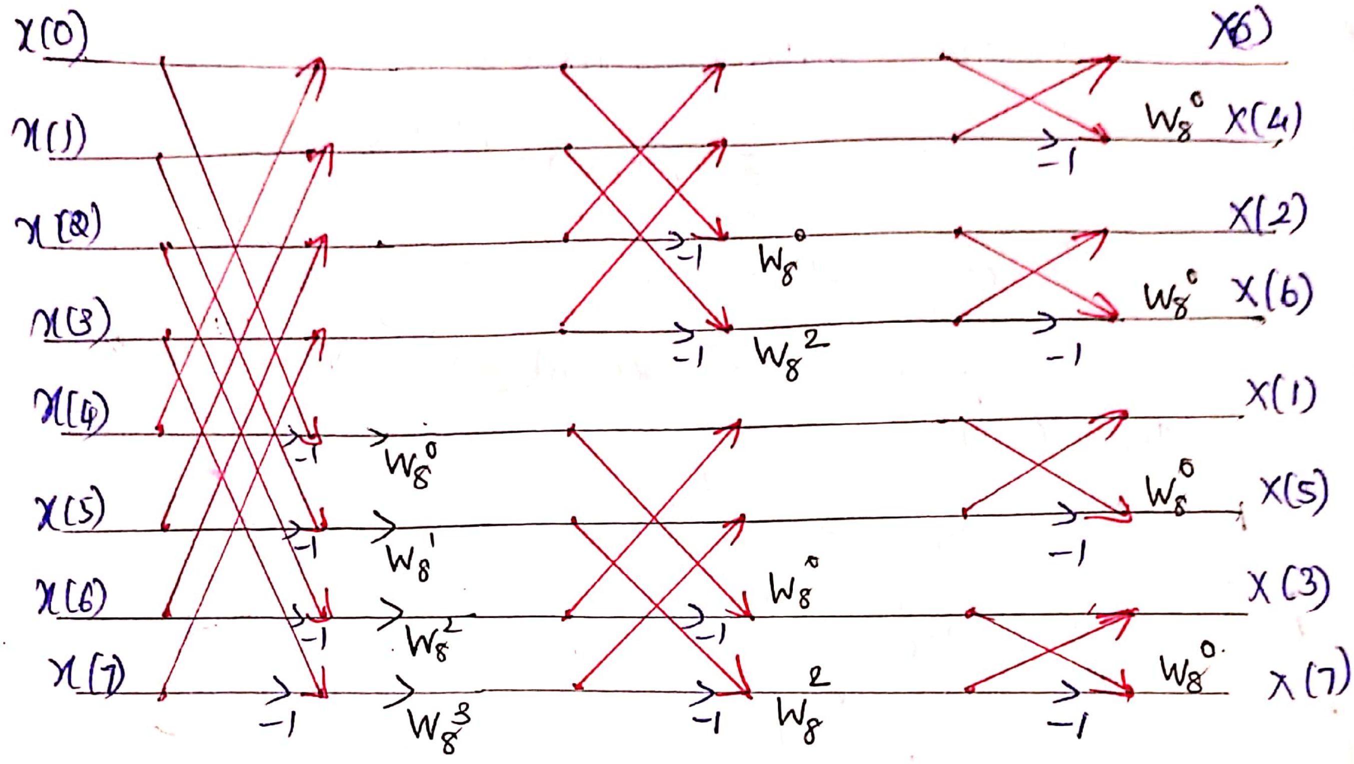
$k=3$, $W_8^3 = e^{-j\frac{2\pi 3}{8}} = e^{-j\pi 3/4}$

$$W_8^3 = -0.707 - j0.707$$

Basic Butterfly Diagram :-



Complete Butterfly structure for DIF.



EX.1.12. Compute DFT of a sequence $x(n) = n+1$ for $N=8$ using DIF FFT algorithm.

$N=8$.

$n = 0, 1, \ldots N-1$

$n = 0, 1, 2, 3, 4, 5, 6, 7$

$x(0) = 1$, $x(1) = 2$, $x(2) = 3$, $x(3) = 4$, $x(5) = 6$

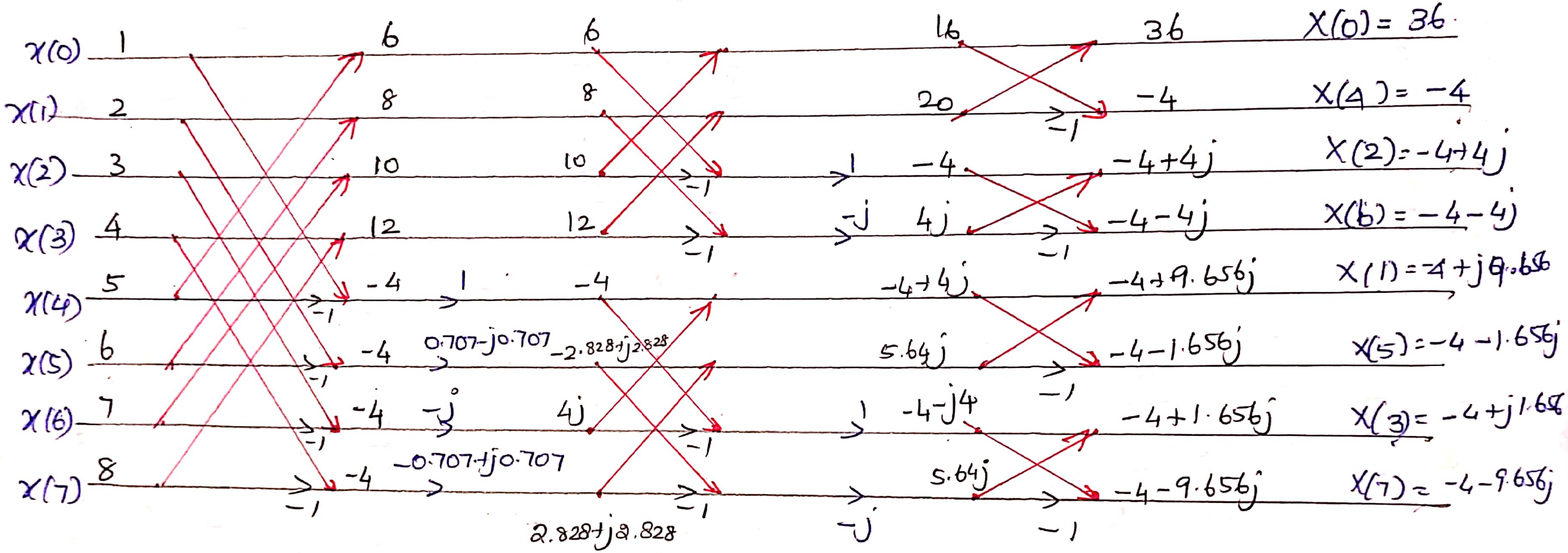
$x(6) = 7$, $x(7) = 8$.

Twiddle factor :-

$$W_N^k = e^{-j2\pi k/N}$$

$W_8^0 = 1$, $W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$, $W_8^3 = -0.707 - j0.707$



$\therefore$ DIF Sequence is , $X(k) = \{36, -4+9.656j, -4+4j, 4+1.656j, -4, -4-j1.656,$

$-4-4j, -4-9.656j\}$

EX. 1.13. Determine 4-point DFT of a sequence $x(n) = \{1, 2, 3, 4\}$ using DIF-FFT algorithm.

$$x(n) = \{1, 2, 3, 4\}$$

Twiddle factor,

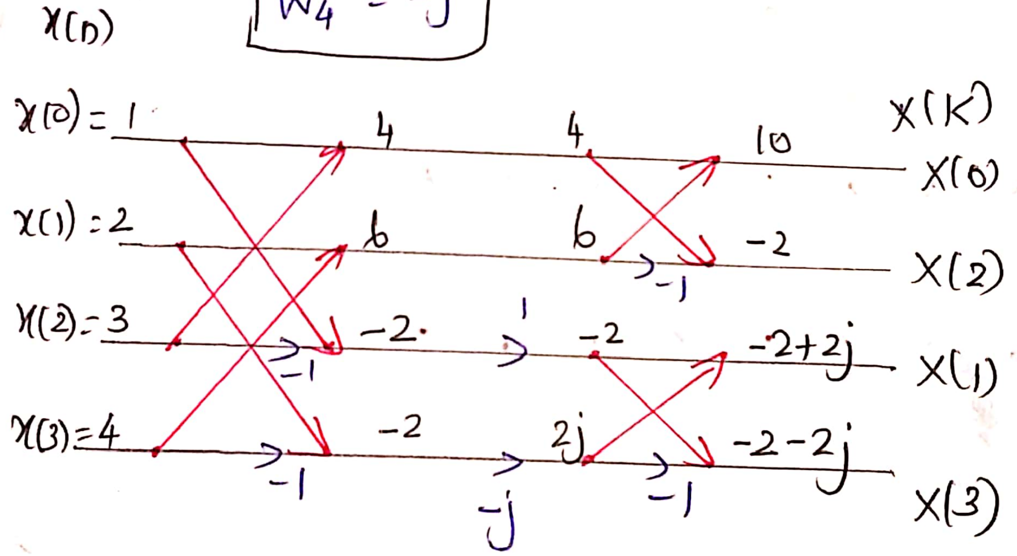
$$W_N^k = e^{-j2\pi k/N}$$

$N = 4$

$k = 0$, $\boxed{W_4^0 = e^0 = 1}$

$k = 1$, $W_4^1 = e^{-j2\pi/4} = e^{-j\pi/2}$

$$\boxed{W_4^1 = -j}$$



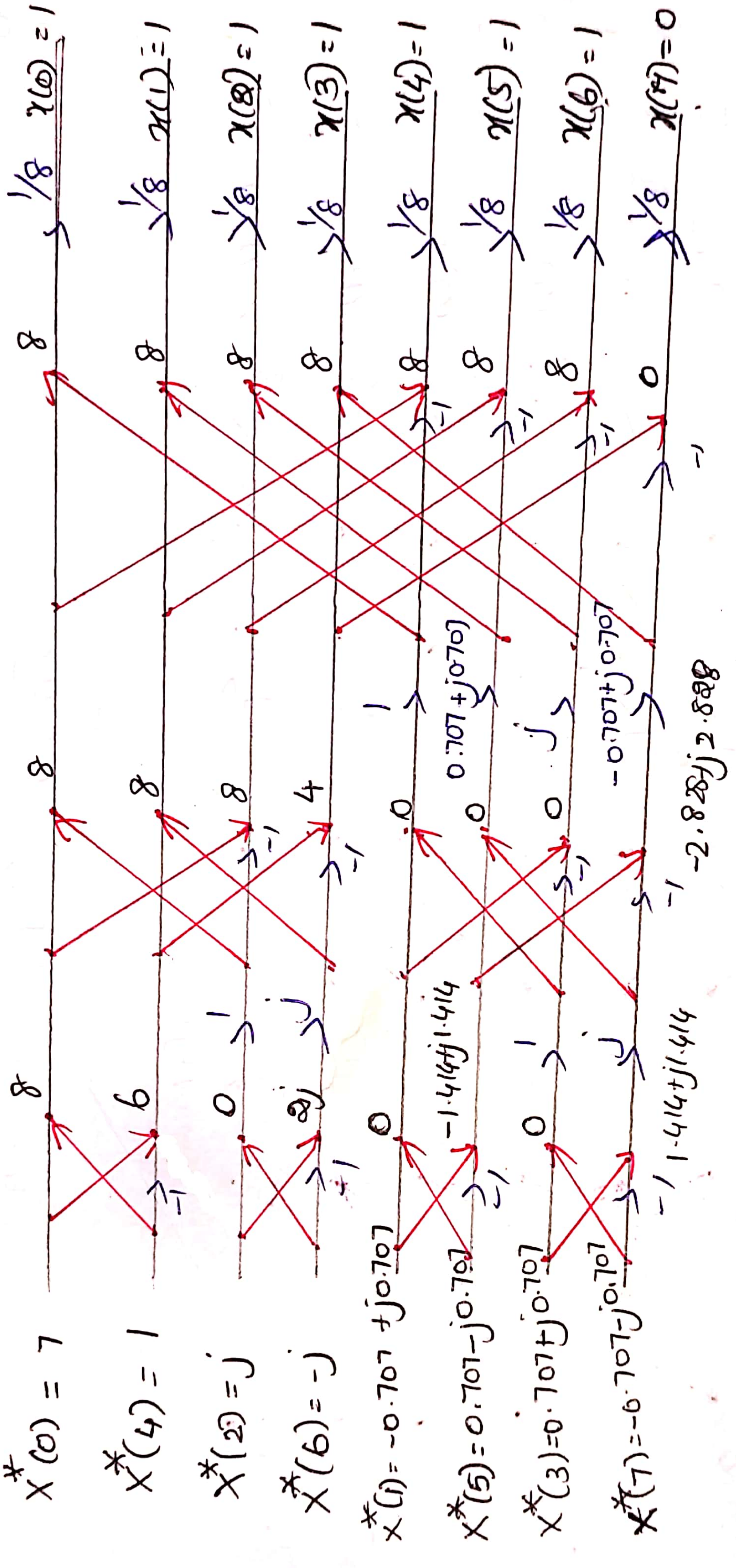
IDFT using FFT algorithm:-

Ex. 1.14. Compute IDFT of the sequence

$$X(K) = \{7, -0.707 - j0.707, -j, 0.707 - j0.707, 1, 0.707 + j0.707, j, -0.707 + j0.707\}$$

using DIT algorithm.

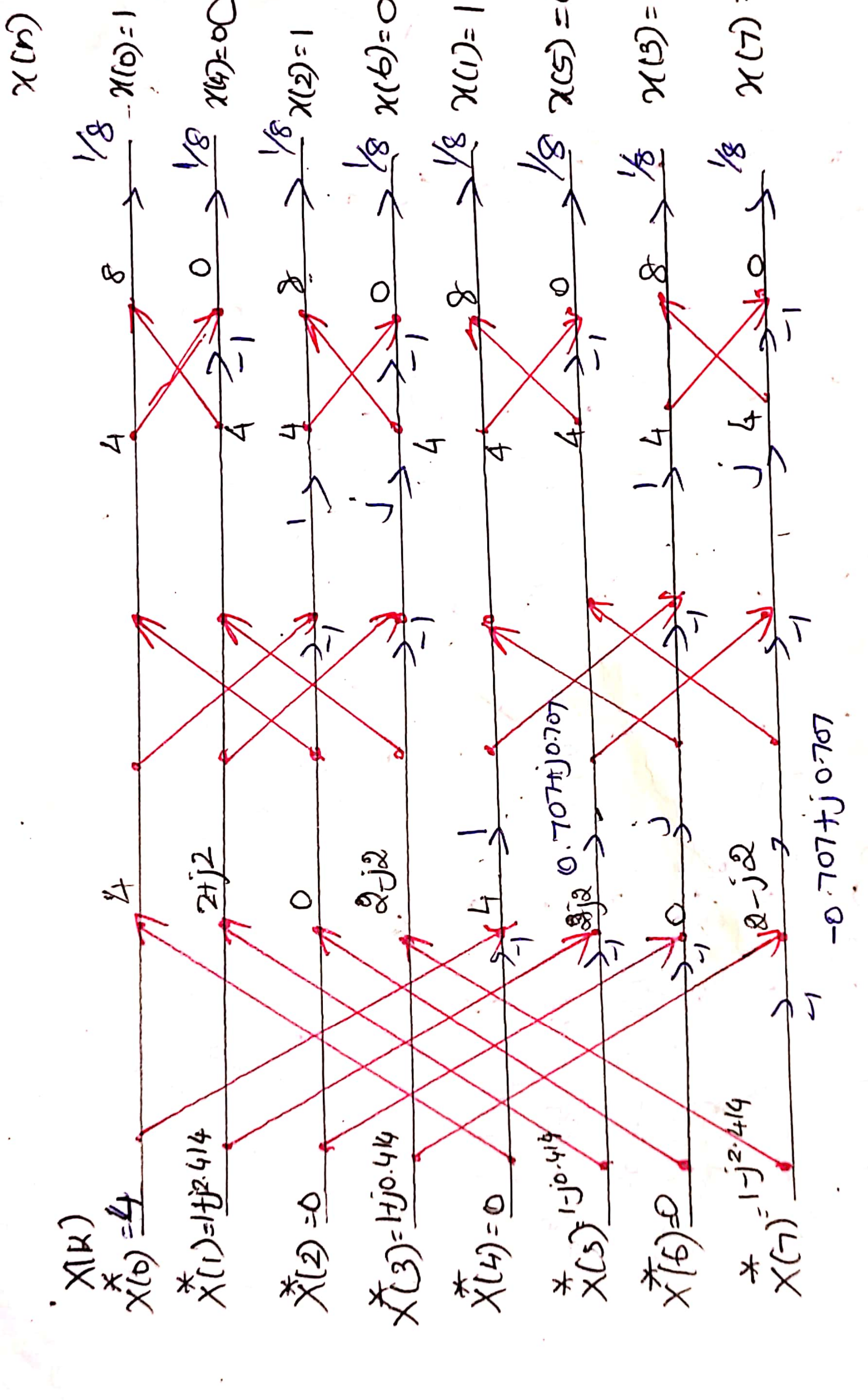
Taking complex conjugate of X(k) and apply bit reversal.



$x(n) = \{1, 1, 1, 1, 1, 1, 1, 0\}$

Ex. 1.15 Find the IDFT of the sequence

$X(k) = \{4, 1, -j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$ using DIF algoritam.



$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

Discrete - Time Fourier transform:-

The DTFT of discrete - time signal $x(n)$ is,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

The IDTFT of $X(e^{j\omega})$ is,

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

1. Find the fourier transform of the following.

a) $\delta(n)$

$$x(n) = \delta(n)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n}$$

$\delta(n) = 0$ for $n \neq 0$
1 for $n = 0$

$$= 1$$

$$F[\delta(n)] = 1$$

b) $x(n) = u(n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} e^{-j\omega n} = 1$$

$$X(e^{j\omega}) = 1 + e^{-j\omega} + e^{-j2\omega} + \dots + \infty$$

$$= \frac{1}{1-e^{-j\omega}}$$

$u(n) = 0$ for $n \leq 0$
1 for $n \geq 0$

$$1 + r + r^2 + \dots \infty = \frac{1}{1-r}$$

$$F[u(n)] = \frac{1}{1-e^{-j\omega}}$$

c) $\delta(n-k)$

$$x(n) = \delta(n-k)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \delta(n-k) e^{-j\omega n}$$

$$X(e^{j\omega}) = e^{-j\omega k}$$

$\delta(n-k) = 1$ for $n = k$
$= 0$ for $n \neq k$

d) $u(n-k)$

$$x(n) = u(n-k)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} u(n-k) e^{-j\omega n}$$

$$= \sum_{n=k}^{\infty} e^{-j\omega n}$$

$$= e^{-j\omega k} + e^{-j\omega(k+1)} + \dots$$

$$= e^{-j\omega k}\left(1 + e^{-j\omega} + e^{-j2\omega} + \dots\right)$$

$$= e^{-j\omega k} \cdot \frac{1}{1-e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{e^{-j\omega k}}{1-e^{-j\omega}}$$

Derivation of DFT from DTFT:

The DFT of $x(n)$ is obtained by sampling one period DTFT $X(e^{j\omega})$ at a finite number of frequency points.

The sampling frequency points are,

$$\omega_k = \frac{2\pi k}{N} \quad ; \text{ for } k = 0, 1, 2, \ldots N-1$$

where $N \rightarrow$ total no. of samples,

The sampling of $X(e^{j\omega})$ is mathematically expressed as,

$$X(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} \quad ; \text{ for } k = 0, 1, \ldots N-1$$

$\therefore$ N-point DFT of $x(n)$ is,

$$DFT[x(n)] = X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \quad ; \text{ for } k = 0, 1, 2, \ldots N-1$$

ie) $X(k) = \{X(0), X(1), X(2), \ldots X(N-1)\}$

The Inverse DFT of $X(k)$ of length N is,

$$IDFT[X(k)] = x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{+j\frac{2\pi kn}{N}} \quad ; \text{ for } n = 0, 1, \ldots N-1$$

1) Compute DFT of $x(n) = \{1, 1, -2, -2\}$

Ans: $\{-2, 3-3j, 0, 3+3j$

2) Find IDFT of $Y(K) = \{1, 0, 1, 0\}$

$$y(n) = \frac{1}{N} \sum_{K=0}^{N-1} Y(K) e^{j2\pi kn/N} \quad n = 0, 1, \ldots N-1$$

Ans: $[0.5, 0, 0.5, 0]$

3) Find DFT of $\{1, 1, 0, 0\}$

Ans: $X(k) = \{2, 1-j, 0, 1+j\}$

4) DFT of $(-1)^n$ for $N = 4$

$x(n) = (-1)^n$

$x(0) = 1, \quad x(1) = -1, \; x(2) = 1, \; x(3) = -1$

Ans: $X(K) = \{0, 0, 4, 0\}$

5) Comput 4-point DFT of the sequence

$$x(n) = 6 + \sin\left(\frac{2\pi n}{N}\right) \; ; n = 0, 1, \ldots N-1$$

$x(0) = 6, \quad x(1) = 7, \; x(2) = 6, \; x(3) = 5$

Ans: $X(K) = \{24, -j2, 0, j2\}$

Ex.1. Find IDFT of the sequence.

$$X(K) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$$

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j2\pi kn/N} \qquad n = 0, 1, \ldots, N-1$$

for $N = 8$,

$$x(n) = \frac{1}{8} \sum_{k=0}^{N-1} X(k)\, e^{j\pi kn/4} \qquad n = 0, 1, \ldots, 7$$

For $n = 0$,

$$x(0) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\right]$$

$$= \frac{1}{8}\left[5 + 0 + 1 - j + 0 + 1 + 0 + 1 + j + 0\right]$$

$$x(0) = 1$$

$$x(1) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j\pi K/4}\right]$$

$$= \frac{1}{8}\left[5 + (1-j)j + 1(-1) + (1+j)(-j)\right]$$

$$= \frac{1}{8}[6]$$

$$x(1) = 0.75$$

$$x(2) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j\pi K/2}\right]$$

$$= \frac{1}{8}\left[5 + (1-j)(-1) + 1 \times 1 + (1+j)(-1)\right]$$

$$x(2) = \frac{1}{8}[4] = 0.5$$

$$x(3) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j3\pi K/4}\right]$$

$$= \frac{1}{8}\left[5 + (1-j)(-j) + 1(-1) + (1+j)(j)\right]$$

$$= \frac{1}{8}(2)$$

$$x(3) = 0.25$$

$$x(4) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j5\pi K/4}\right]$$

$$= 1$$

$$x(5) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j5\pi K/4}\right]$$

$$= \frac{1}{8}\left[5 + (1-j)j + (1)(-1) + (1+j)(-j)\right]$$

$$= \frac{1}{8}[6] = 0.75$$

$$x(6) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j3\pi K/2}\right]$$

$$= \frac{1}{8}\left[5 + (1-j)(-1) + 1(1) + (1+j)(-1)\right]$$

$$= \frac{1}{8}[4] = 0.5$$

$$x(7) = \frac{1}{8}\left[\sum_{K=0}^{7} X(K)\, e^{j7\pi K/4}\right]$$

$$= \frac{1}{8}\left[5 + (1-j)(-j) + 1(-1) + (1+j)(j)\right]$$

$$= 1/8\,(2) = 0.25$$

$$x(n) = \{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$$

• Ex. Determine the 8-point DFT of the sequence :-

$$x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad ; k = 0, 1, \ldots N-1$$

for $N = 8$,

$$X(k) = \sum_{n=0}^{7} x(n)\, e^{-j\pi nk/4} \quad ; k = 0, 1, \ldots 7$$

Ans :- $X(k) = \{6, -0.707 - j1.707, 1 - j, 0.707 + j0.293, 0, 0.707 - j0.293, 1 + j, -0.707 + j1.707\}$

Properties of DFT :-

(i) periodicity

(ii) Symmetry

(iii) Circular Convolution.

(i) Periodicity

1) obtain the circular convolution of the following sequences.

$x(n) = \{1, 2, 1\}$. $h(n) = \{1, -2, 2\}$

$$\underset{x(n)}{\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}} \underset{h(n)}{\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}} = \underset{y(n)}{\begin{bmatrix} 1-2+4=3 \\ 2-2+2=2 \\ 1-4+2=-1 \end{bmatrix}}$$

$y(n) = \{3, 2, -1\}$

Linear convolution:-

no. of samples in the output is $N = L+M-1$

Circular convolution:-

No. of samples in the output $N = \max(L, M)$

H.W:

1. Compute linear & circular convolution of two sequences. $x_1(n) = \{1, 1, 2, 2\}$ and $x_2(n) = \{1, 2, 3, 4\}$

Ans:- for circular $x_3(n) = \{15, 17, 15, 13\}$

1) Find the linear convolution of $x(n) = \{1, -3, 5, -7, 9, -11\}$ with $h(n) = \{-4, 8, -16\}$ using circular convolution.

$x(n) = \{1, -3, 5, -7, 9, -11\}$

ie) $L = 6$

$h(n) = \{-4, 8, -16\}$

ie) $M = 3$

$N = L+M-1 = 6+3-1$

$N = 8$

$x(n) = \{1, -3, 5, -7, 9, -11, 0, 0\}$

$h(n) = \{-4, 8, -16, 0, 0, 0, 0, 0\}$

$$y(n) = \begin{bmatrix} 1 & 0 & 0 & -11 & 9 & -7 & 5 & -3 \\ -3 & 1 & 0 & 0 & -11 & 9 & -7 & 5 \\ 5 & -3 & 1 & 0 & 0 & -11 & 9 & -7 \\ -7 & 5 & -3 & 1 & 0 & 0 & -11 & 9 \\ 9 & -7 & 5 & -3 & 1 & 0 & 0 & -11 \\ -11 & 9 & -7 & 5 & -3 & 1 & 0 & 0 \\ 0 & -11 & 9 & -7 & 5 & -3 & 1 & 0 \\ 0 & 0 & -11 & 9 & -7 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 8 \\ -16 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$y(n) = \{-4, 20, -60, 116, -172, 228, -232, 176\}$

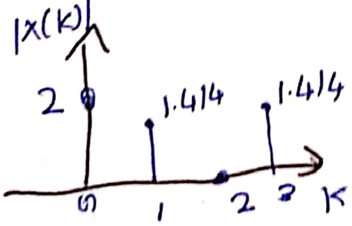
Magnitude & phase spectrum:-

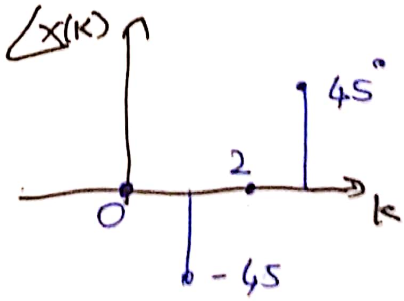
$X(K) = \{2, 1-j, 0, 1+j\}$

$X(K) = \{2\angle 0^\circ, 1.14\angle -45^\circ, 0\angle 0^\circ, 1.414\angle 45^\circ\}$

Magnitude $|X(K)| = \{2, 1.414, 0, 1.414\}$

Phase $\angle X(K) = \{0^\circ, -45^\circ, 0^\circ, 45^\circ\}$





2) Find the 4-point DFT of the sequence $x(n) = n+1$

$N = 4$

$x(n) = n+1,$

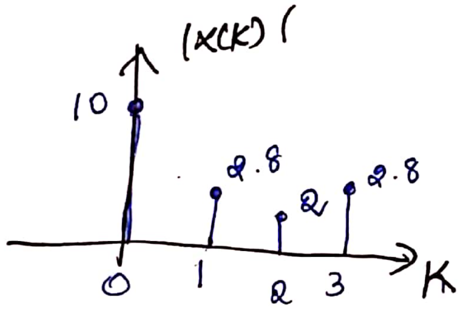
$x(n) = \{1, 2, 3, 4\}$

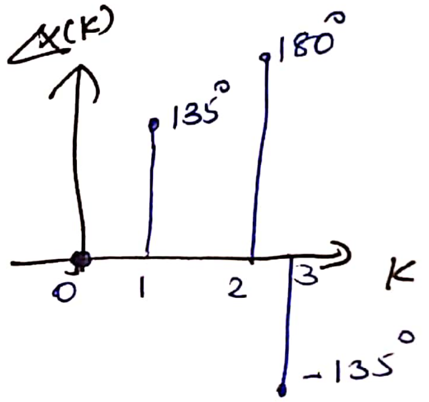
$X(k) = \{10, -2+2j, -2, -2-2j\}$

$X(k) = \{10\angle 0^\circ, 2.8\angle 135^\circ, 2\angle 180^\circ, 2.8\angle -135^\circ\}$

$|X(k)| = \{10, 2.8, 2, 2.8\}$

$\angle X(k) = \{\angle 0^\circ, \angle 135^\circ, \angle 180^\circ, \angle -135^\circ\}$





i) Find the $y(n)$ value for,

$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$, $h(n) = \{1, 1, 1\}$

using overlap add method.

i) Periodicity:

If a sequence $x(n)$ is periodic with Periodicity of N Samples then N-point DFT of the sequence is also periodic with the Periodicity of n samples.

$$x(n+N) = x(n) \quad ; \text{ for all values of } n$$

$$X(K+N) = X(K) \quad ; \text{ for all values of } K.$$

Proof:-

~~into~~ By definition of IDFT,

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi kn}{N}} \quad ; n = 0, 1, \ldots N-1$$

Put $n = n+N$

$$\therefore x(n+N) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi(n+N)K}{N}} \quad ;$$

$$x(n+N) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j\frac{2\pi kn}{N}}\, e^{j\frac{2\pi KN}{N}}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j\frac{2\pi Kn}{N}}\, e^{j2\pi K.} \qquad e^{j2\pi} = 1$$

$$x(n+N) = \frac{1}{N} \sum_{K=0}^{N-1} X(k)\, e^{j\frac{2\pi kn}{N}} \qquad (e^{j2\pi k} = 1^{k} = 1$$

$$\therefore x(n+N) = x(n)$$

By the definition of DFT,

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad ; \quad k=0,1,\ldots N-1$$

Put $k = k+N$,

$$\therefore X(K+N) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n(k+N)/N}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \cdot e^{-j2\pi n}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$X(K+N) = X(K)$$

Hence proved.

(iii) Circular Convolution :-

The DFT of a Circular convolution of two sequences in time domain

If $DFT[x_1(n)] = X_1(K)$ &

$DFT[x_2(n)] = X_2(K)$ then

$$DFT[x_1(n) \circledast x_2(n)] = X_1(K) X_2(K)$$

Proof :-

$$DFT[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$DFT[x_1(n) * x_2(n)] = \sum_{n=0}^{N-1} [x_1(n) * x_2(n)] e^{-j2\pi kn/N}$$

$$x_1(n) * x_2(n) = \sum_{n=0}^{N-1} x_1(m)\, x_2(n-m)$$

$$DFT[x_1(n) * x_2(n)] = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)\, e^{-j\frac{2\pi nk}{N}}$$

$l = n-m$

$n = m+l$

$$= \sum_{m=0}^{N-1} x_1(m) \sum_{l=0}^{N-1} x_2(l)\, e^{-j2\pi(m+l)K/N}$$

$$= \sum_{m=0}^{N-1} x_1(m) \sum_{l=0}^{N-1} x_2(l)\, e^{-j2\pi mk/N} \cdot e^{-j\frac{2\pi lk}{N}}$$

$$= \sum_{m=0}^{N-1} x_1(m)\, e^{-j2\pi mk/N} \sum_{l=0}^{N-1} x_2(l)\, e^{-j\frac{2\pi lk}{N}}$$

$$DFT[x_1(n) * x_2(n)] = X_1(K)\, X_2(K)$$

Symmetry :-

If $x(n)$ is real,

$$X(K) = X^*(N-K)$$

$$X_R(K) = X_R(N-K)$$

$$X_I(K) = -X_I(N-K)$$

$$|X(K)| = |X(N-K)|$$

$$\angle X(K) = -\angle X(N-K)$$

Proof :-

Since $x(n)$ is real, $x^*(n) = x(n)$

then $DFT[x^*(n)] = X(K) = \sum_{k=0}^{N-1} x^*(n)\, e^{-j2\pi kn/N}$

$$= \left(\sum_{K=0}^{N-1} x(n) e^{j2\pi kn/N} \right)^*$$

$$= \left[\sum_{K=0}^{N-1} x(n) e^{j2\pi kn/N} e^{-j2\pi Nn/N} \right]^*$$

$$= \left[\sum_{K=0}^{N-1} x(n) e^{-j2\pi(N-k)n/N} \right]^*$$

$$X(K) = X^*(N-K)$$

$$\therefore X(k) = X^*(N-k)$$

$$X(K) = X_R(K) + j X_I(K)$$

$$= \left[X_R(N-k) + j X_I(N-k) \right]^*$$

$$X(K) = X_R(N-K) - j X_I(N-K)$$

$$\Rightarrow X_R(K) = X_R(N-K)$$

$$X_I(K) = - X_I(N-K)$$

$$|X(K)| = \sqrt{X_R(K)^2 + X_I(K)^2} = \sqrt{X_R(N-K)^2 + X_I(N-K)^2}$$

$$|X(N-K)| = \sqrt{X_R(N-k)^2 + X_I^2(N-K)}$$

$$\therefore |X(K)| = |X(N-K)|$$

$$|||^{rly}, \quad \angle X(K) = -\angle X(N-K)$$