JEPPIAAR INSTITUTE OF TECHNOLOGY

“Self-Belief | Self Discipline | Self Respect”

DEPARTMENT

OF

ELECTRONICS AND COMMUNICATION ENGINEERING

LECTURE NOTES

EC8391 – CONTROL SYSTEMS ENGINEERING

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Prepared by

Mr.A.Antony Charles

Assistant Professor/EEE

UNIT I – SYSTEM COMPONENTS AND THEIR REPESENTATION

Control System: Terminology and Basic Structure-Feed forward and Feedback control theory Electrical and Mechanical Transfer Function Models-Block diagram Models-Signal flow graphs models-DC and AC servo Systems-Synchronous -Multivariable control system

A system is arrangement of components or devices connected together to perform a specific function. A control system is a type of system, which for a specific input gives corresponding output.

Definition:

- When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a **system**
- When a output quantity is controlled by varying the input quantity, the system is called **control system**

Example: Automatic Tea/Coffee Maker, Electric Hand Drier, Automatic Electric Iron, Servo Voltage Stabilizer, Missile launching systems etc

BASIC STRUCTURE OF A SYSTEM

The system consists of various components such as

Input: flow of energy or material that causes process to react or respond. **Manipulated Input** is a input which is subjected to control. **Disturbance Input** is an undesirable and unavoidable input to the plant, also known as Disturbance or Noise.

Command Input: The external input which is independent of the feedback control. **Reference Input Element:** This element estimates the relationship between the command and reference input.

Error Detector: Also known as comparator, it compares the reference input with feedback signal.

Controller: This element is responsible for suitable control action. **Control Signal** is the output of the controller.

Error Signal: Output of error detector.

Final Control Element: Actuator element block.

Controlled System: Process, in which a particular condition is to be controlled. **Disturbance Input:** Variable which designer has no control or little information is available on magnitude or function or time.

Controlled Variable: It is influenced by both manipulated variable and disturbance. **Feedback :** It is a function of controlled variable. It is Used to correct the nonlinear in the controlled system.

CLASSIFICATION OF CONTROL SYSTEM

1. Open loop and closed loop system
2. Linear and nonlinear system
3. Time Invariant and Time Variant system
4. Continuous and Discrete system
5. SISO and MIMO system.
6. Lumped parameter and Distributed parameter system
7. Deterministic and Stochastic control system
8. Static and Dynamic system

Open loop and closed loop system

Open Loop Control System: A control system in which the control action is totally independent of output of the system. Any physical system which does not automatically correct the variation in its output or control system in which the output quantity has no effect upon the input quantity are called open-loop control system. This means that the output is not feedback to the input for correction.

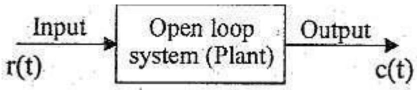


Fig: Open loop system.

In open loop system the output can be varied by varying the input. But due to external disturbances the system output may change. When the output changes due to disturbances, it is not followed by changes in input to correct the output. In open loop systems the changes in output are corrected by changing the input manually.

Example:

1. Light Switch : Lamp glows whenever light switch is „ON" irrespective of light is required or not.
2. Volume of Stereo System: Volume is adjusted manually irrespective of output volume level.
3. Man walking on road with closed eyes. It is very difficult to walk on the desired path.
4. Electric hand drier
5. Automatic washing machine
6. Bread toaster

Merits:

1. System are simple in construction and design.
2. Easy to maintain
3. Economical
4. Stable

1. Systems are inaccurate and not reliable
2. Recalibration of the controller is necessary time to time
3. Changes in output due to external disturbance are not corrected automatically.

Closed Loop Control System: Control systems in which the output has an effect upon the input quantity in order to maintain the desired output value are called closed loop systems.

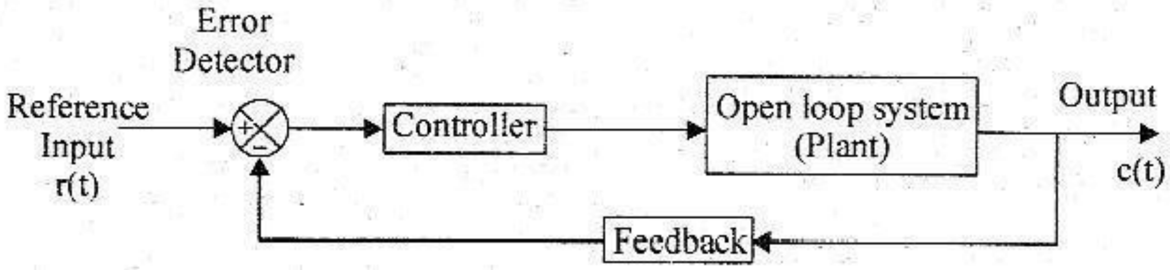


Fig: Closed loop control system.

The open loop system can be modified as closed loop system by providing a feedback. The provision of feedback automatically corrects the changes in output due to disturbances. Hence the closed loop system is also called automatic control system. The general block diagram of an automatic control system is shown in fig.

It consists of an error detector, a controller, plant (open loop system) and feedback path elements. The reference signal (or input signal) corresponds to desired output. The feedback path elements samples the output and converts it to a signal of same type as that of reference signal. The feedback signal is proportional to output signal and it is fed to the error detector.

The error signal generated by the error detector is the difference between reference signal and feedback signal. The controller modifies and amplifies the error signal to produce better control action. The modified error signal is fed to the plant to correct its output.

Example:

1. Automatic electric Iron: Heating element are controlled by output temperature of the iron.
2. Air Conditioner: It"s function depends on the temperature of the room.
3. Water Level Controller: Input water is controlled by water level of the reservoir.
4. Man walking with eyes open in a road, eye performs as error detector, compares actual path of the movement with prescribed path and generates error signal. This error signal transmits the corresponding control signal to the legs to connect the actual movement to desired path.

Merits:

1. The closed loop systems are accurate and reliable
2. Reduced effect of Nonlinearity and disturbance.
3. Operating frequency zone is high.
4. Senses the environmental changes and external disturbance and accordingly takes necessary control action.

Demerits:

1. The closed loop systems are complex and costly.
2. The feedback in closed loop system may lead to oscillatory response.
3. The feedback reduces the overall gain of the system.

SNO	**Open Loop Control System**	**Closed Loop Control System**
1.	Feedback is absent	Feedback is always present
2.	An error detector is not present	An error detector is always present
3.	Stable	It may become unstable
4.	Easy to construct	Complicated in construction
5.	It is an economical	It is costly.
6.	Has small bandwidth	Has large bandwidth
7.	It is inaccurate.	It is accurate
8.	Less maintenance	More maintenance
9.	It is unreliable	It is reliable
10.	Examples: Hand drier, tea maker	Examples: Servo voltage stabilizer, perspiration

Linear and Nonlinear system: Linear system obeys law of superposition. The principle of superposition states that the response produced by simultaneous application of two different forcing functions is the sum of individual responses.

If r(t) is input and y(t) is the output, and $r_1(t) \rightarrow y_1(t)$ for $t \geq 0$ and $r_2(t) \rightarrow y_2(t)$ for $t \geq 0$, if the input $r(t)= ar_1(t)+br_2(t)$: for $t \geq 0$,

then for a linear system the output must be $y(t)= ay_1(t)+by_2(t)$, for $t \geq 0$.

Time Invariant and Time Variant system:

For a time invariant system the parameter does not vary with time, response of such system is independent of time at which input is applied.

For time variant the response depends on time. For example, in the space vehicle control system the mass of the vehicle reduces as time increases and fuel decreases.

Continuous and Discrete system

If all the elements of the describing equation is define for all time, then the system is continuous time (Differential Equation).If as in sampled date system, some elemental equation are define or used only at discrete time points, then the system is discrete time system (Difference Equation).

SISO and MIMO system

Single input – Single output and Multi input and multi output system.

Lumped parameter and Distributed parameter system

In Lumped Parameter system the significant variable of the system are lumped at some discrete point, hence they are described by ordinary differential equation. When the significant variables are distributed with respect to space and time, they are described by partial fraction with time, with variables as independent variables.

Deterministic and Stochastic control system

In deterministic system the response is predictable, whereas in Stochastic system the variables and parameters are random and the response is not predicable.

Static and Dynamic system

In a dynamic system the present output depends on present and past inputs. In a static system the present output depends on only the present input.

MATHEMATICAL MODELS

Mathematical modelling of any control system is the first and foremost task that a control engineer has to accomplish for design and analysis of any control engineering problem. It is nothing but the process or technique to express the system by a set of mathematical equations (algebraic or differential in nature).

Analysis means the process of finding the response or output of a system when it is excited by an input or excitation provided we know the mathematical model of the system. On the other hand, **design or synthesis** means we have to find out the system equations or the arrangement of the components, provided we know the output of the system for an input.

Commonly used mathematical models are-

1. Differential equation model.
2. State space model.
3. Transfer function model.

Transfer Function. The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

$$G(s) = \frac{\mathscr{L}[\text{output}]}{\mathscr{L}[\text{input}]}\bigg|_{\text{zero initial conditions}}$$

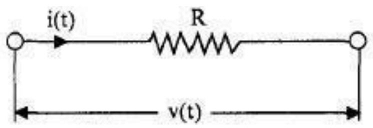
The applicability of the concept of the transfer function is limited to linear, time-invariant, differential equation systems. The transfer function approach, however, is extensively used in the analysis and designs of such systems are as follows.

1. The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.
2. The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.
3. The transfer function includes the units necessary to relate the input to the output; however, it does not provide any information concerning the physical structure of the system. (The transfer functions of many physically different systems can be identical.)
4. If the transfer function of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.
5. If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system. Once established, a transfer function gives a full description of the dynamic characteristics of the system, as distinct from its physical description.

ELECTRICAL AND MECHANICAL SYSTEMS Electrical Systems:

Most of the electrical systems can be modelled by three basic elements : Resistor, inductor, and capacitor. Circuits consisting of these three elements are analysed by using Kirchhoff's Voltage law and Current law.

Resistor*:* The circuit model of resistor is shown in Fig.

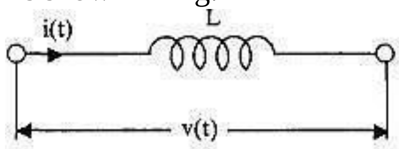


The mathematical model is given by the Ohm's law relationship,

$$V(t) = i(t)\,R$$

$$i(t) = V(t)/R$$

Inductor: The circuit representation is shown in Fig.



The input output relations are given by Faraday's law,

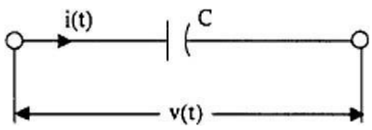
$$V(t) = L\, di(t)/dt$$

$$i(t) = (1/L)\int v\, dt$$

where Integral of v dt is known as the flux linkages. Thus

$$I(t) = \psi(t)/ L$$

Capacitor: The circuit symbol of a capacitor is given in Fig.



$$v(t) = (1/C)\int i\, dt$$

$$i(t) = C\, dv/dt$$

In eqn. idt is known as the charge on the capacitor and is denoted by *'q'*. Thus

$$q = \int i\, dt$$

$$v(t) = q(t)/C$$

Mechanical System

There are two types of mechanical systems based on the type of motion.

- Translational mechanical systems
- Rotational mechanical systems

Modeling of Translational Mechanical Systems

Translational mechanical systems move along a **straight line**. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.

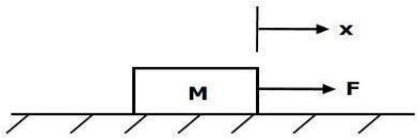
If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system. Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

Mass

Mass is the property of a body, which stores **kinetic energy**. If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and frictions are negligible.

$F_{m \alpha a}$

$$\mathrm{F} = \mathrm{Fm} = \mathrm{Ma} = M\,\frac{d^2\mathrm{X}}{dt^2}$$



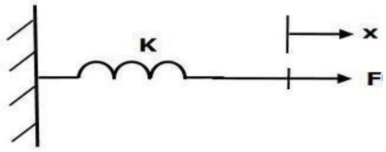
- **F** is the applied force, $\mathbf{F_m}$ is the opposing force due to mass, **M** is mass, **a** is acceleration
- **x** is displacement

Spring

Spring is an element, which stores **potential energy**. If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring. This opposing force is proportional to the displacement of the spring. Assume mass and friction arenegligible.

$F_{k \alpha x}$

$F = F_{k} = KX$



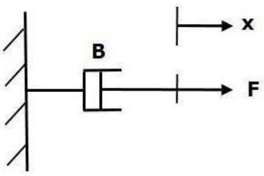
F is the applied force, $\mathbf{F_k}$ is the opposing force due to elasticity of spring, **K** is spring constant

- **x** is displacement

Dashpot

If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body.
Assume mass and elasticity are negligible.

$F_{b \alpha w}$; $F = F_b = B\frac{d\mathrm{X}}{dt}$;



- **Zb** is the opposing force due to friction of dashpot, **B** is the frictional coefficient, **v** is velocity
- **x** is displacement

Modeling of Rotational Mechanical Systems

Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.

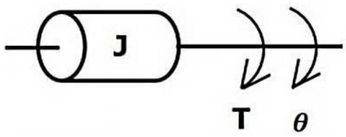
If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied

torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.

If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$T_j \alpha\ \alpha;\ T = T_j = J\alpha = J\frac{d^2\theta}{dt^2}$

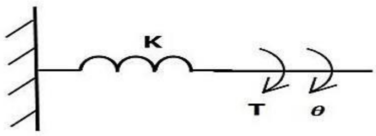
- **T** is the applied torque, $\mathbf{T_j}$ is the opposing torque due to moment of inertia, **J** is moment of inertia
- **α** is angular acceleration, **θ** is angular displacement

Torsional Spring

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.

If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.

$T_k \alpha\ \theta;\ T = T_k = K\theta$

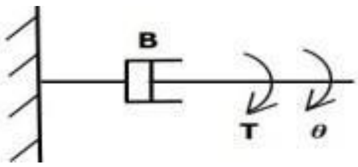


- **T** is the applied torque, $\mathbf{T_k}$ is the opposing torque due to elasticity of torsional spring, **K** is the torsional spring constant, **θ** is angular displacement

Dashpot

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot. This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.

$T_b \alpha\ \theta;\ T = T_b = B\omega = B\frac{d\theta}{dt}$



- $\mathbf{T_b}$ is the opposing torque due to the rotational friction of the dashpot, **B** is the rotational friction coefficient, **ω** is the angular velocity, **θ** is the angular displacement

Two systems are said to be **analogous** to each other if the following two conditions are satisfied.

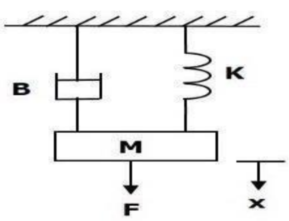
- The two systems are physically different
- Differential equation modelling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of **translational mechanical system** are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure.

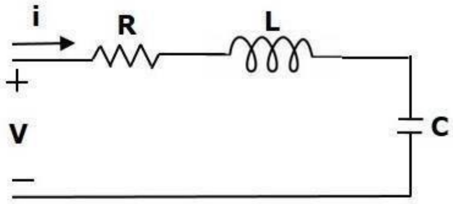


The **force balanced equation** for this system is

$$F = f_m + f_b + f_k = 0$$

$$F = M\frac{d^2K}{dt^2} + B\frac{dK}{dt} + Kx \quad --- (1)$$

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series. The input voltage applied to this circuit is VV volts and the current flowing through the circuit is ii Amps.



Mesh equation for this circuit is
V=Ri+Ldi/dt+1/c∫idt------**(2)**
Substitute, i=dq/dt in Equation 2.

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + + \frac{q}{c}$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + + \frac{1}{c}q \quad --- (3)$$

By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

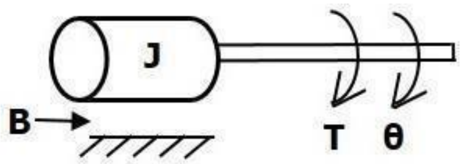
Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance (1c)(1c)
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Similarly, there is torque voltage analogy for rotational mechanical systems. Let us now discuss about this analogy.

Torque Voltage Analogy

In this analogy, the mathematical equations of **rotational mechanical system** are compared with mesh equations of the electrical system.

Rotational mechanical system is shown in the following figure.



The torque balanced equation is

$$T = T_{j1} + T_b + T_k$$

$$T = J\frac{d^2\theta}{dt^2} + B_1\frac{d\theta}{dt} + K\theta \quad --- (4)$$

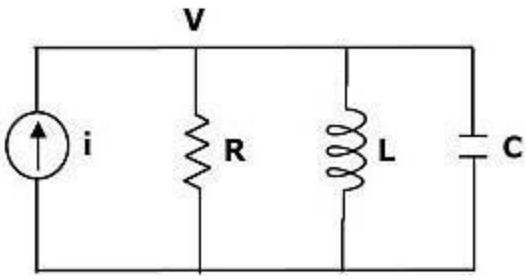
By comparing Equation 4 and Equation 3, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1c)(1c)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the **translational mechanical system** are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$$\frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} = i(t) \quad --- (5)$$

Substitute, V=dΨ/dt in Equation 5.

$$i(t) = \frac{1}{R}\frac{df}{dt} + \frac{1}{L}f + C\frac{d^2f}{dt^2}$$

$$i(t) = C\frac{d^2f}{dt^2} + \frac{1}{R}\frac{df}{dt} + \frac{1}{L}f \quad --- 6$$

(By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance(1R)(1R)
Spring constant(K)	Reciprocal of Inductance(1L)(1L)
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems. Let us now discuss this analogy.

Torque Current Analogy

In this analogy, the mathematical equations of the **rotational mechanical system** are compared with the nodal mesh equations of the electrical system.

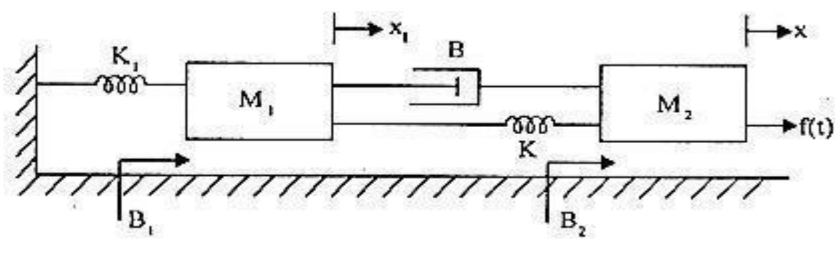
By comparing Equation 4 and Equation 6, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance(1R)(1R)
Torsional spring constant(K)	Reciprocal of Inductance(1L)(1L)
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)

These analogies are helpful to study and analyze the non-electrical system like mechanical system from analogous electrical system.

Examples

1. Write the differential equations governing the mechanical system shown in fig .And determine the transfer function?



Solution

In the given system, applied force f(t) is the input and displacement X is the output

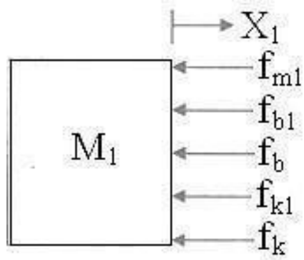
Let, Laplace transfer of f(t)=£{f(t) = F(s)}

Laplace transfer of x =)=£{X} = X(s)

Laplace transfer of X_1= £{X_1} = X_1(s)

Hence the required transfer function is $\frac{X(s)}{F(s)}$

At Node 1 (M1)



$$f_{m1} = M_1\frac{d^2X_1}{dt^2};\ f_{b1} = B_1\frac{dX_1}{dt};\ f_{k1} = K_1X_1;$$

$$f_b = B\frac{d}{dt}(X_1 - X);\ f_k = K(X_1 - X);$$

By Newton's second law, $f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0$

$$\therefore M_1\frac{d^2X_1}{dt^2} + B_1\frac{dX_1}{dt} + B\frac{d}{dt}(X_1 - X) + K_1X_1 + K(X_1 - X);$$

On taking Laplace transform of above equation with zero initial conditions we get,

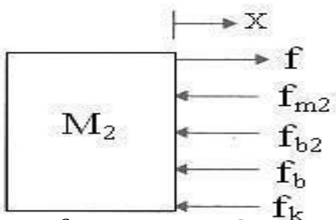
$$M_1s^2X_1(s) + B_1sX_1(s) + Bs[X_1(s) - X(s)] + K_1X_1(s) + K[X_1(s) - X(s)] = 0$$

$$X_1(s)[M_1s^2 + (B_1 + B)s + (K_1 + K)] - X(s)[Bs + K] = 0$$

$$X_1(s)[M_1s^2 + (B_1 + B)s + (K_1 + K)] = X(s)[Bs + K]$$

$$\therefore X_1(s) = X(s)\frac{Bs + K}{M_1s^2 + (B_1 + B)s + (K_1 + K)} \quad \text{.........................}(1)$$

At Node 2 (M2)



$$f_{m2} = M_2\frac{d^2X}{dt^2};\ f_{b2} = B_2\frac{dX}{dt};\ f_b = B\frac{d}{dt}(X - X_1);\ f_k = K(X - X_1)$$

By Newton's second law, $f_{m2} + f_{b2} + f_b + f_k = f(t)$

$$M_2\frac{d^2X}{dt^2} + B_2\frac{dX}{dt} + B\frac{d}{dt}(X - X_1) + K(X - X_1) = f(t)$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$M_2s^2X(s) + B_2sX(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)] = F(s)$$

$$X(s)[M_2s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s) \quad \text{.........................}(2)$$

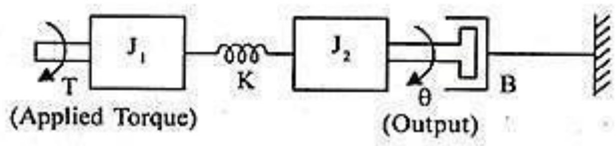
Substituting $X_1(s)$ from equation (1) in equation (2) we get,

$$X(s)[M_2s^2 + (B_2 + B)s + K] - \frac{(Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$X(s) * \frac{[M_1s^2 + (B_2 + B)s + K][M_2s^2 + (B_1 + B)s + (K_1 + K)] - (Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{M_1s^2 + (B_1 + B)s + (K_1 + K)}{[M_1s^2 + (B_1 + B)s + (K_1 + K)][M_2s^2 + (B_1 + B)s +] - (Bs + K)^2}$$

2. Write the differential equations governing the mechanical rotational system as shown in fig, obtain the transfer function of the system.



SOLUTION

In the given system, applied force f(t) is the input and displacement X is the output.

Let, Laplace transfer of T = L(T)=T(s)

Laplace transfer of θ =L(θ)=Θ(s)

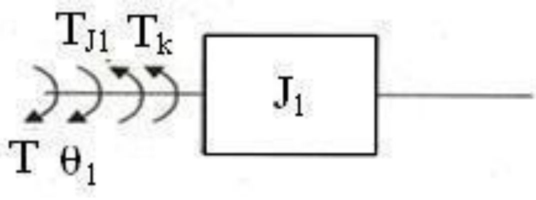
Laplace transfer of θ_1=L(θ1)=Θ1(s)

Hence the required transfer function is= $\frac{\theta(s)}{T(s)}$

The system has two nodes and they are mass $J_1$and J_2, the differential equations governing the system are given by torques balance equations at these nodes.

Let the displacement of mass J_1 be $_1$ .The free body diagram of J_1 is shown in fig. the opposing forces acting on $J_1$are marked as T_j, and T_k.

Free body diagram-1



$$T_{j1} = J_1\frac{d^2\theta_1}{dt^2};\ T_k = K(\theta_1 - \theta)$$

By Newton's second law,$T_{j1} + T_k = T$

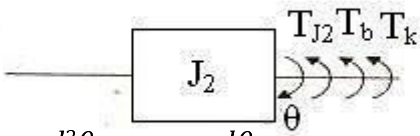
$$J_1\frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

On taking Laplace transform of above equation we get,

$$J_1s^2\theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$(J_1s^2 + K)\theta_1(s) - K\theta(s) = T(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Free body diagram-2



$$T_{j2} = J_2\frac{d^2\theta}{dt^2}; T_b = B\frac{d\theta}{dt}; \ T_k = K(\theta - \theta_1)$$

By Newton's second law, $T_{j2} + T_b + T_k = 0$

$$J_2\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

On taking Laplace transform of above equation we get,

$$J_2s^2\theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$(J_2s^2 + Bs + K)\theta(s) - \theta_1(s) = 0$$

$$\theta_1(s) = \frac{(J_2s^2 + Bs + K)}{K}\theta(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2)$$

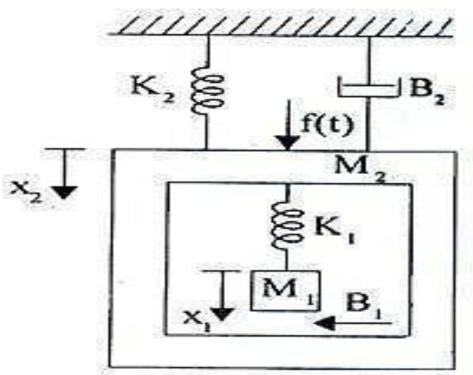
Substitute $\theta_1(s)$ from equation 2 in equation 1 we get,

$$(J_1s^2 + K)\frac{(J_2s^2 + Bs + K)}{K}\theta(s) - K\theta(s) = T(s)$$

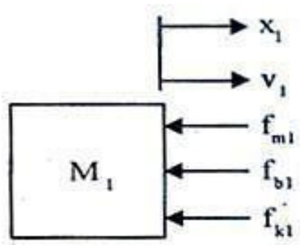
$$*\ \frac{(J_1s^2 + K) + (J_2s^2 + Bs + K) - K^2}{K} + \theta(s) = T(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{K}{(J_1s^2 + K) + (J_2s^2 + Bs + K) - K^2}}$$

3. Write the differential equations governing the mechanical system shown in fig. draw the force current electrical analogous circuit.

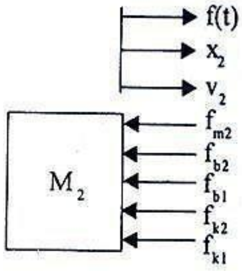


Solution:
Node-1



$$M_1\frac{d^2X_1}{dt^2} + B_1\frac{d(X_1 - X_2)}{dt} + K_1(X_1 - X_2) = 0$$

Node-2



$$M_2 \frac{d^2X_2}{dt^2} + B_1 \frac{d(X_2 - X_1)}{dt} + B_2 \frac{dX_2}{dt} + K_1(X_2 - X_1) + K_2X_2 = f(t)$$

Force- Current analogous circuits

The electrical analogous is given by:

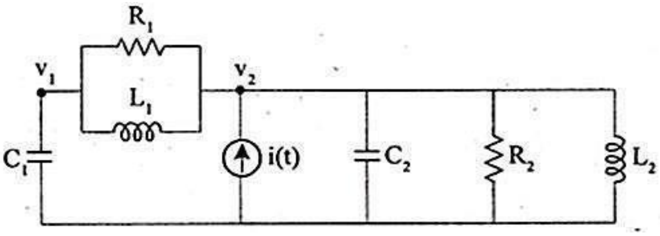
$f(t) = i(t)$	$M_1 \rightarrow C_1$	$B_1 \rightarrow 1/R_1$	$K_1 \rightarrow 1/L_1$
$v_1 \rightarrow v_1$	$M_2 \rightarrow C_2$	$B_2 \rightarrow 1/R_2$	$K_2 \rightarrow 1/L_2$
$v_2 \rightarrow v_2$	$B_{12} \rightarrow 1/R_{12}$		

Thus the systems equations are:

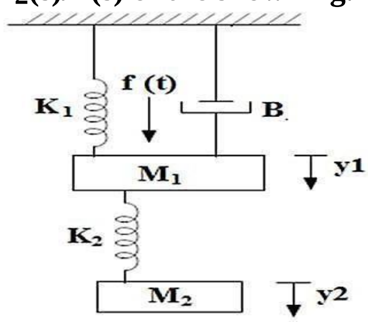
$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1}(v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)\, dt = 0 \quad \ldots\ldots (1)$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{L_2}\int v_2\, dt + \frac{1}{R_1}(v_2 - v_1) + \frac{1}{L_1}\int (v_2 - v_1)\, dt = i(t) \quad \ldots\ldots (2)$$

And the circuit is given by



4.Determine the transfer function $Y_2(s)/F(s)$ of the shown fig.



Solution:

Let Laplace transform of f(t) = L{f(t)} = F(s)

Let Laplace transform of y_1 = L{y_1} = $Y_1(s)$

Let Laplace transform of y_2 = L{y_2} = $Y_2(s)$

The system has two nodes and they are mass M_1 and M_2.

The differential equations governing the system are the force balance equations of at these nodes.

Consider Mass M1,

Free Body diagram of M1 ,

$f(t) = M_1 \frac{d^2F_1}{dt^2} + B\frac{dF1}{dt} + K_2(Y_1 - Y_2) + K_1Y_1$ ------ 1

on taking Laplace Transform of equation (1) wit
h zero initial conditions,

$M_1S^2Y_1(s) + BsY_1(s) + K_1Y_1(s) + K_2\,[Y_1(s) - Y_2(s)] = F(s)$

$Y_1(s)[M_1S^2 + Bs + (K_1+K_2)] - Y_2(s)K_2 = F(s)$ -------------------------- 2

Consider Mass M_2,

$$M_2\, d^2Y_2/dt^2 + K_2\,[Y_2 - Y_1] = 0$$

On taking Laplace Transform of equation (3) with zero initial conditions,

$M_2S^2Y_2(s) + K_2\,[Y_2(s) - Y_1(s)] = 0$--4

$Y_2(s)\,[M_2S^2 + K_2] - K_2\,Y_1(s) = 0;$

$Y_1(s) = Y_2(s)\,[M_2S^2 + K_2] / K_2$

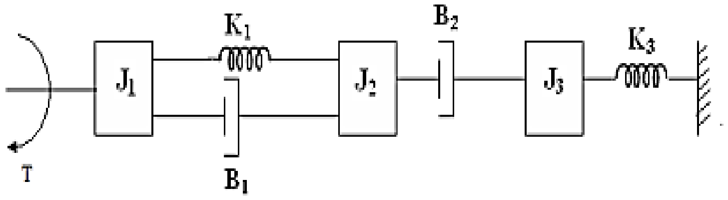
Substituting equation $Y_1(s)$ from equation (5) into equation (2) we get,

$Y_2(s)\,[M_2S^2 + K_2] / K_2 + [M_1S^2 + Bs + (K_1+K_2)] - K_2\,Y_2(s) = F(s)$

$Y_2(s)\,([M_2S^2 + K_2] + [M_1S^2 + Bs + (K_1+K_2)] - K_2^2\,) / K_2 = F(s)$

$$Y_2(s) / F(s) = K_2 / ([M_2S^2 + K_2] + [M_1S^2 + Bs + (K_1+K_2)] - K_2^2)$$

5.Write the differential equations governing the mechanical rotational system shown in fig. Draw the torque-voltage and torque-current electrical analogous circuits and verify by writing mesh and node equations.



Solution:

The given mechanical rotational system has three nodes. The differential equations governing the mechanical rotational system are given by torque balance equations at these nodes.

Let the angular displacements J_1, J_2 and J_3 be θ_1, θ_2 and θ_3 respectively. The corresponding angular velocities be ω_1, ω_2 and ω_3

Consider J_1.

By Newton"s second law we get

$T_{j1} = J_1\frac{d^2\theta_1}{dt^2}$; $T_{b1} = B_1\frac{d(\theta_1 - \theta_2)}{dt}$; $T_{k1} = K_1(\theta_1 - \theta_2)$

By Newtons second law $T_{j1} + T_{b1} + T_{k1} = T$

$$J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d(\theta_1 - \theta_2)}{dt} + K_1(\theta_1 - \theta_2) = T \text{............................(1)}$$

Consider J_2

$T_{j2} = J_2\frac{d^2\theta_2}{dt^2}$; $T_{b2} = B_2\frac{d(\theta_2 - \theta_3)}{dt}$;$T_{k1} = K_1(\theta_2 - \theta_1)$; $T_{b1} = B_1\frac{d(\theta_2 - \theta_1)}{dt}$

By Newtons second law, $T_{j2} + T_{b2} + T_{k1} + T_{b1} = 0$

$$J_2\frac{d^2\theta_2}{dt^2} + \frac{d(\theta_2 - \theta_3)}{dt} + K_1(\theta_2 - \theta_1) + B_1\frac{d(\theta_2 - \theta_1)}{dt} = 0 \text{............(2)}$$

Consider J_3

$$T_{j3} = J_3\frac{d^2\theta_3}{dt^2};\ T_{b2} = B_2\frac{d(\theta_3 - \theta_2)}{dt};\ T_{k3} = K_3\,\theta_3$$

By Newtons Second law, $T_{j3} + T_{b2} + T_{k3} = 0$

$$J_3\frac{d^2\theta_3}{dt^2} + B_2\frac{d(\theta_3 - \theta_2)}{dt} + K_3\,\theta_3 = 0 \text{...} (3)$$

On replacing the angular displacement by angular velocity in the differential equations we get

$$\therefore \left(\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt};\ \frac{d\theta}{dt} = \omega \text{ and } \theta = \int \omega\, dt\right)$$

$$J_1\frac{d\omega_1}{dt} + B_1(\omega_1 - \omega_2) + K_1\int(\omega_1 - \omega_2)dt = T$$

$$J_2\frac{d\omega_2}{dt} + B_1(\omega_2 - \omega_1) + B_2(\omega_2 - \omega_3) + K_1\int(\omega_2 - \omega_1)\,dt = 0$$

$$J_3\frac{d\omega_3}{dt} + B_2(\omega_3 - \omega_2) + K_3\int \omega_3\ dt = 0$$

Torque voltage analogous circuit

The electrical analogous elements for the elements of mechanical rotational systems are given below

$\omega_1 \rightarrow i_1$ $J_1 \rightarrow L_1$ $B_1 \rightarrow R_1$ $K_1 \rightarrow 1/C_1$

$\omega_1 \rightarrow i_2$ $J_2 \rightarrow L_2$ $B_2 \rightarrow R_2$ $K_3 \rightarrow 1/C_3$

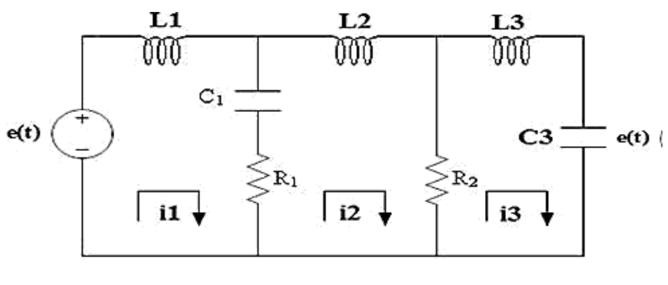
$\omega_3 \rightarrow i_3$ $J_3 \rightarrow L_3$

The Mesh basis equations using Kirchhoff‟s voltage law for the circuit is given by

$$L_1\frac{di_1}{dt} + R_1(i_1 - i_2) + \frac{1}{C_1}\int(i_1 - i_2)\,dt = e(t)$$

$$L_2\frac{di_2}{dt} + R_1(i_2 - i_1) + R_2(i_2 - i_3) + \frac{1}{C_1}\int(i_2 - i_1)\,dt = e(t)$$

The electrical circuit is given by



Torque current analogous circuit

The electrical analogous elements for the elements of mechanical rotational systems are

$T \rightarrow i(t)$ $\omega_1 \rightarrow v_1$ $J_1 \rightarrow C_1$ $B_1 = 1/R_1$ $K_1 \rightarrow 1/L_1$

$\omega_2 \rightarrow v_2$ $J_2 \rightarrow C_2$ $B_2 = 1/R_2$ $K_3 \rightarrow 1/L_3$

$\omega_3 \rightarrow v_3$ $J_3 \rightarrow C_3$

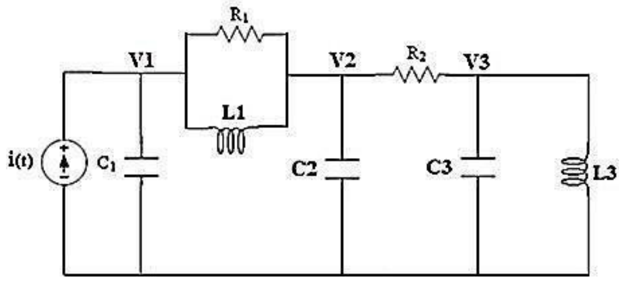
The node basis equations using Kirchhoff‟s current law for the circuit is

$$C_1\frac{dv_1}{dt} + \frac{1}{R_1}(v_1 - v_2) + \frac{1}{L_1}\int(v_1 - v_2)\,dt = i(t) \ \text{......} (1)$$

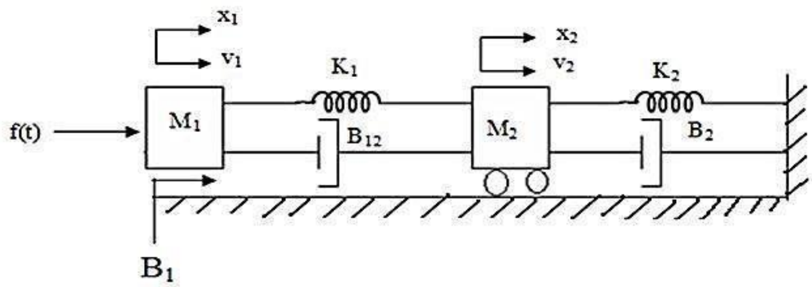
$$C_2\frac{dv_2}{dt} + \frac{1}{R_1}(v_2 - v_1) + \frac{1}{R_2}(v_2 - v_3) + \frac{1}{L_1}\int(v_2 - v_1)\,dt = 0 \ \text{......} (2)$$

$$C_3 \frac{dv_3}{dt} + \frac{1}{R_2}(v_3 - v_2) + \frac{1}{L_3}\int v_3\, dt = 0 \quad \ldots\ldots(3)$$

The electrical circuit is given by



6.Write the differential equations governing the mechanical system shown in fig. Draw the force-voltage and force current electrical analogous circuits and verify by writing mesh and node equations.



Solution:

At M1:

$$f_{m1} = M_1 \frac{d^2X_1}{dt^2}\ ;\ f_{b1} = B_1\frac{dX_1}{dt}\ ;\ f_{b12} = B_{12}\frac{d}{dt}(X_1 - X_2)\ ;\ f_{k1} = K_1(X_1 - X_2)$$

$$M_1\frac{d^2X_1}{dt^2} + B_1\frac{dX_1}{dt} + B_{12}\frac{d}{dt}(X_1 - X_2) + K_1(X_1 - X_2) = 0$$

At M2:

$$M_2\frac{d^2X_2}{dt^2} + B_2\frac{dX_2}{dt} + B_{12}\frac{d}{dt}(X_2 - X_1) + K_2X_2 + K_1(X_2 - X_1) = 0$$

Replacing the displacement by velocity in the differential equation we get,

$$\frac{d^2X}{dt^2} = \frac{dv}{dt}\ ;\ \frac{dX}{dt} = v\ ;\ X = \int v\, dt$$

$$M_1\frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + \int K_1(X_1 - X_2)\, dt = f(t)$$

$$M_2\frac{dv_2}{dt} + B_2 v_2 + B_{12}(v_2 - v_1) + \int K_2(V_2 - V_1)\, dt = 0$$

Force voltage analogous circuit

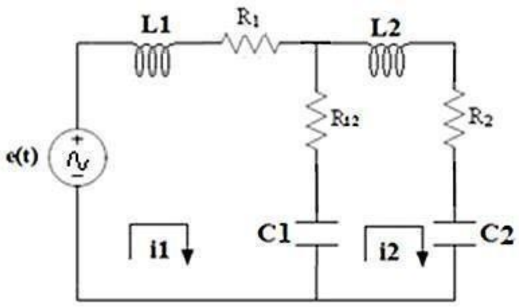
The electrical analogous elements for the elements of mechanical system is given by

f(t) = e(t)	$M_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow 1/C_1$
$v_1 = i_1$	$M_2 \rightarrow L_2$	$B_2 \rightarrow R_2$	$K_2 \rightarrow 1/C_2$
		$B_{12} \rightarrow R_{12}$	

The mesh basis equations using Kirchhoff‟s voltage law for the circuit shown is

$$L_1\frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int(i_1 - i_2)\, dt = e(t)$$

$$L_2\frac{di_2}{dt} + R_2 i_2 + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int(i_2 - i_1)\, dt = 0$$



Force current analogous circuit

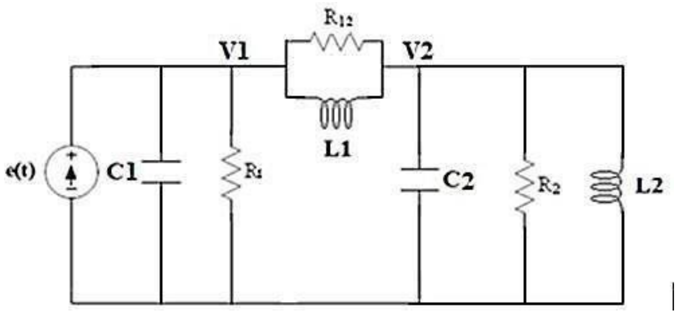
The electrical analogous elements for the elements of mechanical system is given by

f(t) = i(t)	$M_1 \rightarrow C_1$	$B_1 \rightarrow 1/R_1$	$K_1 \rightarrow 1/L_1$
$v_1 \rightarrow v_1$	$M_2 \rightarrow C_2$	$B_2 \rightarrow 1/R_2$	$K_2 \rightarrow 1/L_2$
$v_2 \rightarrow v_2$	$B_{12} \rightarrow 1/R_{12}$		

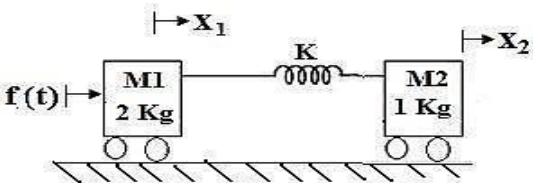
The node basis equations using Kirchhoff's current law for the circuit is

$$C_1\frac{dv_1}{dt} + \frac{1}{R_1}v_1 + \frac{1}{R_{12}}(v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)\,dt = i(t) \quad \ldots\ldots(1)$$

$$C_2\frac{dv_2}{dt} + \frac{1}{R_2}v_2 + \frac{1}{R_{12}}(v_2 - v_1) + \frac{1}{L_2}\int v_2\,dt + \frac{1}{L_1}\int (v_2 - v_1)\,dt = 0 \quad \ldots\ldots(2)$$



7.Derive the transfer function of the system show in the figure.



Applying Newton second law at M_1 ,

$$M_1\frac{d^2X_1}{dt^2} + K(X_1 - X_2) = F(t)$$

Taking Laplace transform on both sides,

$$M_1s^2X_1(s) + K[X_1(s) - X_2(s)] = F(s)$$

$$X_1(s)[M_1s^2 + K] - KX_2(s) = F(s)$$

Applying Newton second law at M_2

$$M_2\frac{d^2X_2}{dt^2} + K(X_2 - X_1) = 0$$

Taking Laplace transform on both sides,

$$M_2s^2X_2(s) + K[X_2(s) - X_1(s)] = 0$$

$$X_2(s)[M_2s^2 + K] - KX_1(s) = 0$$

$$X_2(s)[M_2s^2 + K] = KX_1(s)$$

$$X_1(s) = \frac{X_2(s)[M_2s^2 + K]}{K}$$

substitute value of $x_1(s)$ in (1)

$$\frac{X_2(s)[M_2s^2+K]}{K}[M_1s^2+K] - KX_2(s) = F(s)$$

$$X_2(s)\left(\frac{[M_2s^2+K][M_1s^2+K]}{K} - K\right) = F(s)$$

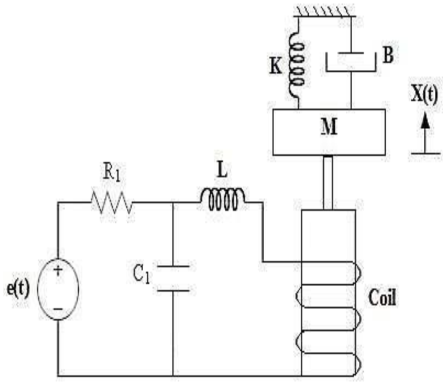
$$X_2(s)\left(\frac{[M_2s^2+K][M_1s^2+K]-K^2}{K}\right) = F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{K}{[M_2s^2+K][M_1s^2+K]-K^2}$$

Substituting the value of M1 =2kg and M2=1kg we get

$$\frac{X_2(s)}{F(s)} = \frac{K}{(S^2+K)(2S^2+K)-K^2}$$

8. In the system shown in the fig below, R, L, C are electric parameters while K, M, B are mechanical parameters. Find the transfer function X(s)/E1(s) for the system where E1(t) is input voltage while X(t) is the output displacement.



Apply Kirchhoff‟s voltage law at loop 1 in the above fig we get

$$Ri_1 + \frac{1}{C}\int(i_1 - i_2)\,dt = e(t)$$

Taking laplace Transform of the above equation

$RI_1(s) + \frac{1}{Cs}[I_1(s) - I_2(s)] = E(s)$ (or)

$(R+\frac{1}{Cs})I_1(s) - \frac{1}{Cs}[I_2(s)] = E(s) --- (1)$

Apply Kirchoff's Voltage law at loop 2 we get

$$L\frac{di_2}{dt} + \frac{1}{C}\int(i_2 - i_1)\,dt = -e_b = -K\frac{dx}{dt}$$

Taking laplace Transform of the above equation

$$sLI_2(s) + \frac{1}{Cs}[I_2(s) - I_1(s)] = -sK_b\,X(s)$$

$$\frac{1}{Cs}[I_2(s) - I_1(s)] = -sK_b\,X(s) - sLI_2(s)$$

$$\frac{1}{Cs}[I_1(s) - I_2(s)] = sK_b\,X(s) + sLI_2(s) \quad --- (2)$$

Substituting (2) in (1)

$$RI_1(s) + sK_b\,X(s) + sLI_2(s) = E(s)$$

From equation (2)

$$\frac{1}{Cs}I_1(s) = sK_b\,X(s) + sLI_2(s) + \frac{1}{Cs}I_2(s)$$

$$= sK_b\,X(s) + (sL + \frac{1}{Cs})I_2(s)$$

$$I_1(s) = s^2K_b\,C\,X(s) + (s^2LC + 1)I_2(s) \quad --- (3)$$

Substituting (3) in (1)

$$(R + \frac{1}{Cs})\,[(s^2LC + 1)I_2(s) + s^2K_b\,C\,X(s)] - \frac{1}{Cs}I_2(s) = E(s)$$

Apply Newtons Second law at M

$$F_c = K_c I_2 = M\frac{d^2X}{dt^2} + B\frac{dX}{dt} + 2KX$$

Taking Laplace Transform

$$K_c I_2(s) = [Ms^2 + Bs + 2K]X(s)$$

$$I_2(s) = \frac{[Ms^2 + Bs + 2K]}{K_c}X(s)$$

Substituting this value of I (s) in eqn (3)

$$(s^2LC + sL + R)\frac{[Ms^2 + Bs + 2K]}{K_c}) X(s) + \quad 2K_b RC + sK_b)X(s) = E(s)$$

After simplying the equation we get

$$\frac{X(s)}{E(s)} = \frac{K_c}{[(RLCs^4 + L(M + RCB)s^3 + \{RM + LB + RC(2LK + K_bK_c)\}s^2 + (RB + 2LK + K_bK_c)s + 2RK]}$$

BLOCK DIAGRAMS

Block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

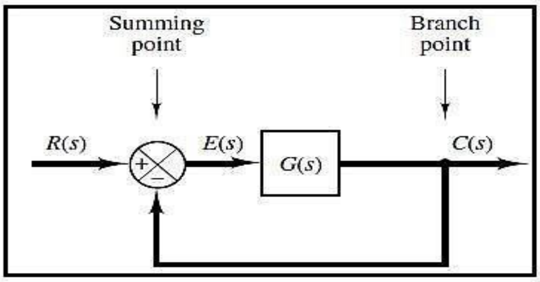
In a block diagram all system variables are linked to each other through functional blocks. The *functional* block or simply *block* is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Note that the signal can pass only in the direction of the arrows. Thus a block diagram of a control system explicitly shows a unilateral property.

Figure above shows an element of the block diagram. The arrowhead pointing toward the block indicates the input, and the arrowhead leading away from the block represents the output. Such arrows are referred to as *signals.*

The dimension of the output signal from the block is the dimension of the input signal multiplied by the dimension of the transfer function in the block. The advantages of the block diagram representation of a system are that it is easy to form the overall block diagram for the entire system by merely connecting the blocks of the components according to the signal flow and that it is possible to evaluate the contribution of each component to the overall performance of the system.

In general, the functional operation of the system can be visualized more readily by examining the block diagram than by examining the physical system itself. A block diagram contains information concerning dynamic behaviour, but it does not include any information on the physical construction of the system. Consequently, many dissimilar and unrelated systems can be represented by the same block diagram.

It should be noted that in a block diagram the main source of energy is not explicitly shown and that the block diagram of a given system is not unique. A number of different block diagrams can be drawn for a system, depending on the point of view of the analysis.

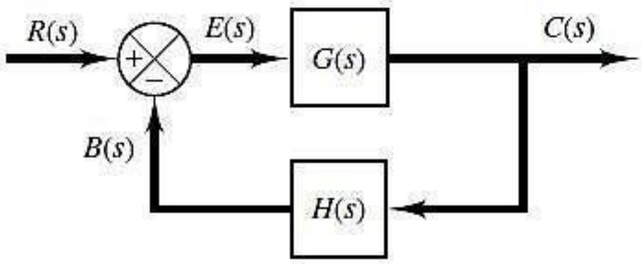


Summing Point. Referring to Figure, a circle with a cross is the symbol that indicates a summing operation. The plus or minus sign at each arrowhead indicates whether that signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

Branch Point. A *branch point* is a point from which the signal from a block goes concurrently to other blocks or summing points.

Block Diagram of a Closed-Loop System. Figure below shows an example of a block diagram of a closed-loop system. The output C(s) is fed back to the summing point, where it is compared with the reference input R(s). The closed-loop nature of the system is clearly indicated by the figure. The output of the block, C(s) in this case, is obtained by multiplying the transfer function G(s) by the input to the block, E(s).Any linear control system may be represented by a block diagram consisting of blocks, summing points, and branch points.

When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of the output signal to that of the input signal. For example, in a temperature control system, the output signal is usually the controlled temperature. The output signal, which has the dimension of temperature, must be converted to a force or position or voltage before it can be compared with the input signal. This conversion is accomplished by the feedback element whose transfer function is H(s). The role of the feedback element is to modify the output before it is compared with the input.

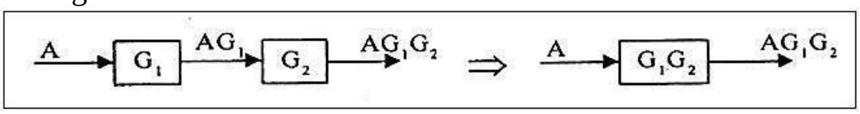


For the system shown in Figure, the output C(s) and input R(s) are related as follows:

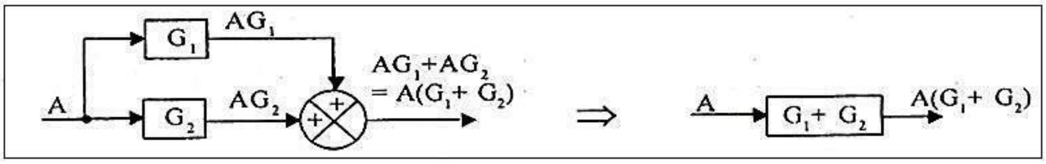
$$C(s) = G(s)E(s)$$
$$E(s) = R(s) - B(s)$$
$$E(s)= R(s) - H(s)C(s)$$
$$C(s) = G(s)[R(s) - H(s)C(s)]$$
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

RULES FOR REDUCTION OF BLOCK DIAGRAM

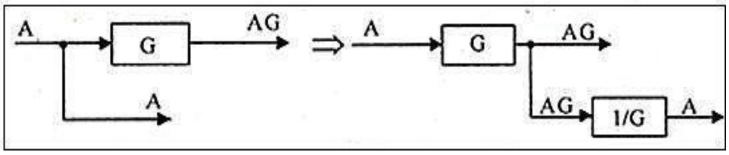
Rule 1: Combining blocks in series or cascade:



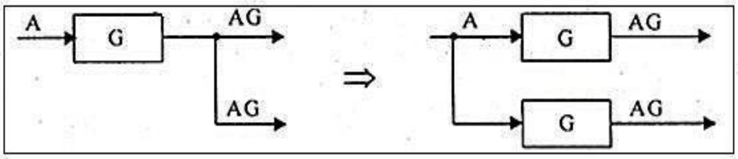
Rule 2: Combining blocks in parallel:



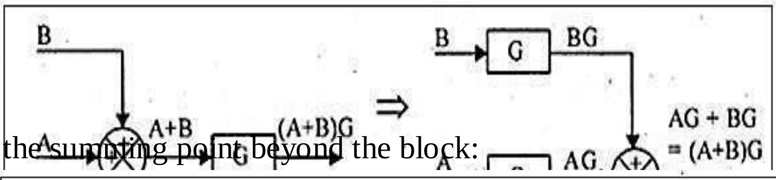
Rule 3: Moving take off (Branch Point) ahead of the block:



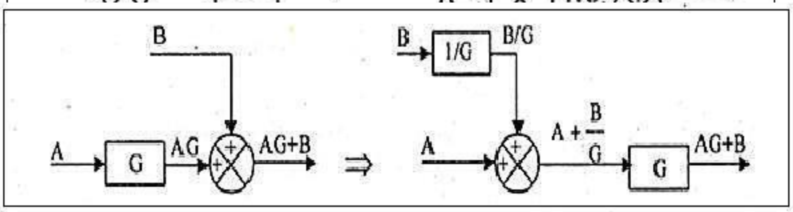
Rule 4: Moving take off (Branch Point) beyond the block:



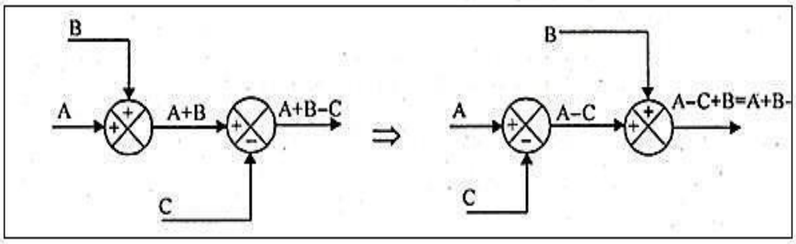
Rule 5: Moving the summing point ahead of the block:



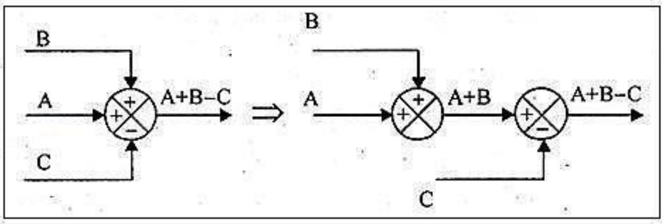
Rule 6: Moving the summing point beyond the block:



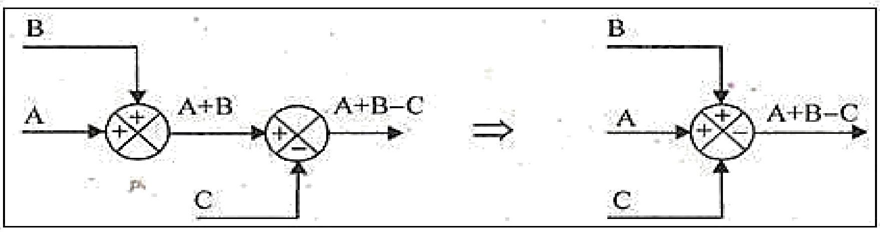
Rule 7: Interchanging Summing Points:



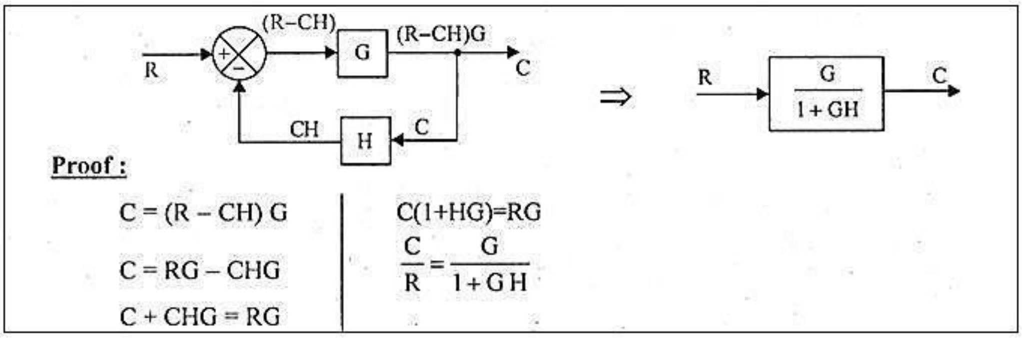
Rule 8: Splitting Summing Points:



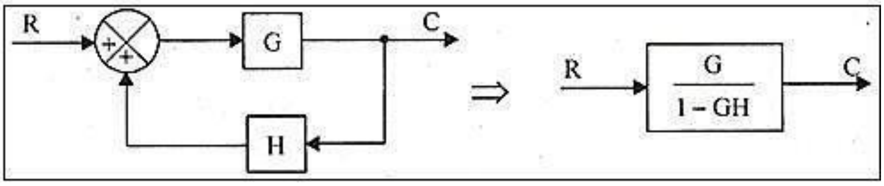
Rule 9: Combining Summing Points:



Rule 10: Eliminating negative feedback:

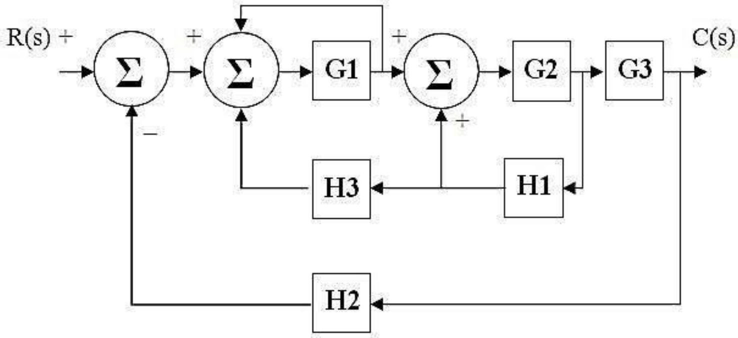


Rule 11: Eliminating positive feedback:

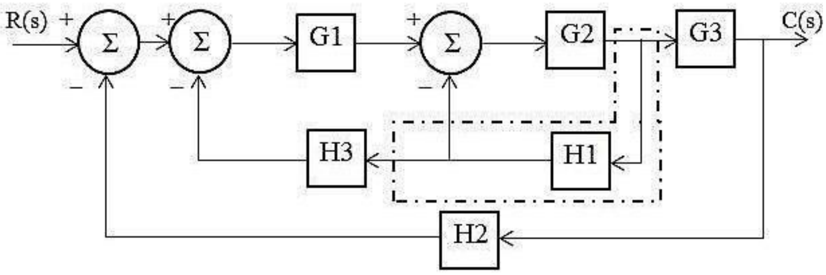


Examples:

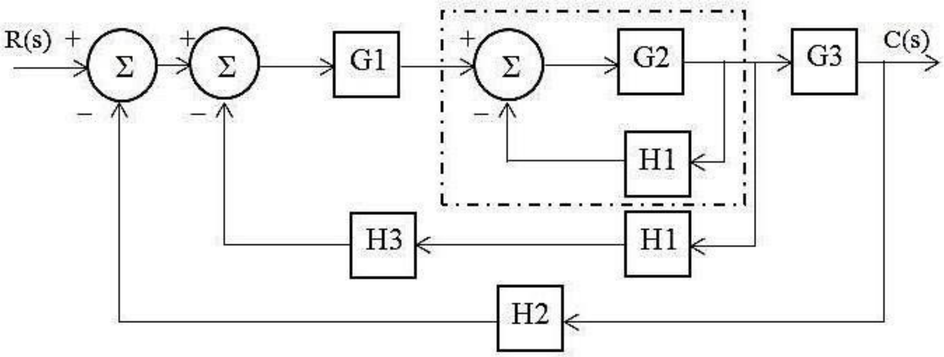
1. Find the transfer function of the system shown in the fig. using block diagram reduction technique and signal flow graph technique.



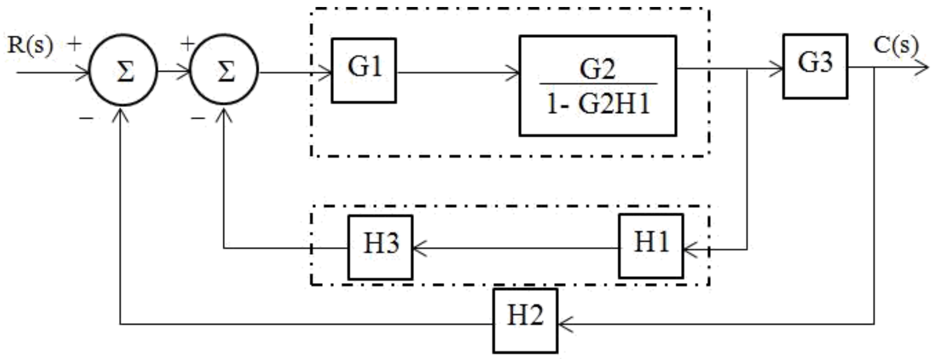
Step 1: Re arranging the branch points.



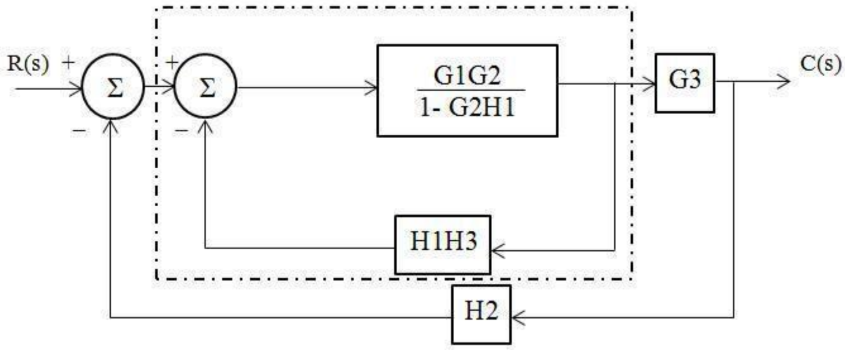
Step 2: Eliminating the feedback paths.



Step 3: Combining the blocks in cascade.



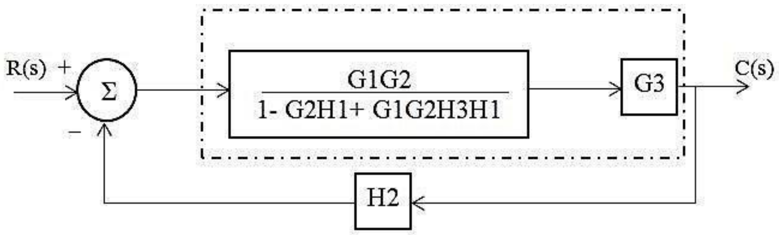
Step 4: Eliminating the feedback path



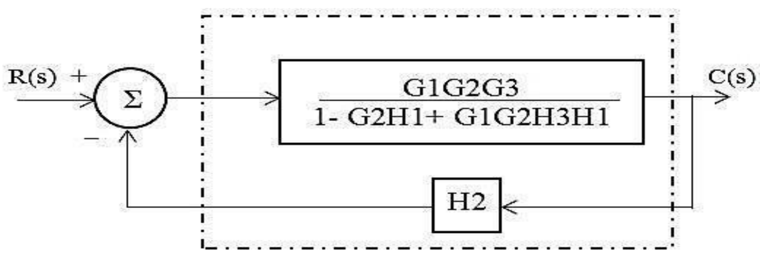
$$= \frac{\dfrac{G2G2}{1- G2H1}}{1+\dfrac{G1G2}{1- G2H1}\times H3H1}$$

$$= \frac{G1G2}{1- G2H1+ G1G2H3H1}$$

Step 5: Combining the blocks in cascade.



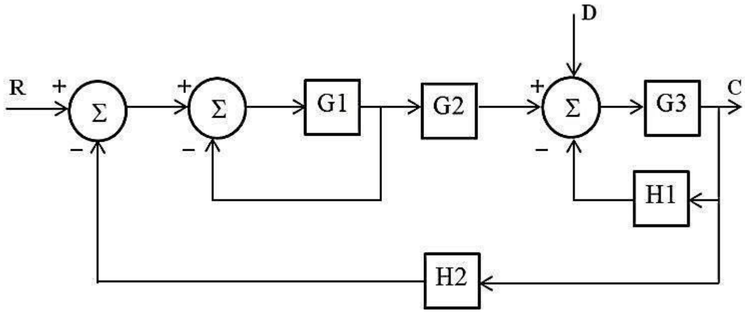
Step 6: Eliminating the feedback path



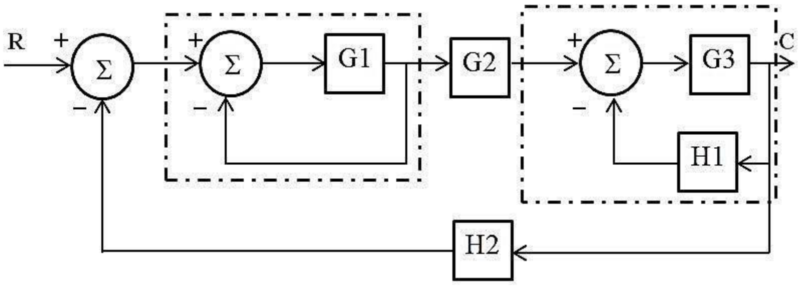
$$= \frac{\dfrac{G1G2G3}{1- G2H1+ G1G2H3H1}}{1+\dfrac{G1G2G3}{G2H1+G1G2H3H1}}$$

$$=\frac{C(s)}{R(s)} = \frac{G1G2G3}{1- \ G2H1+G1G2H3H1+ \ G1G2G3H2}$$

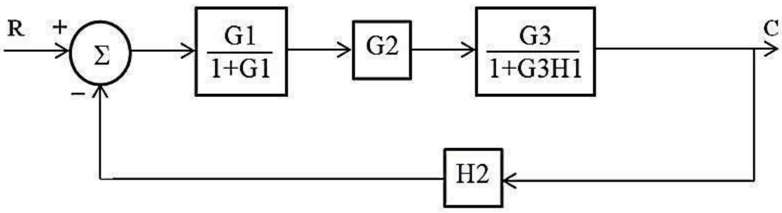
2. For the block diagram shown below, find the output C due to R and disturbance D.



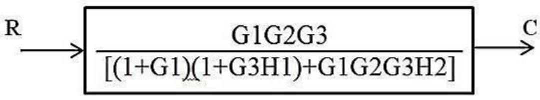
Step 1: Assuming D = 0, the block diagram becomes as shown in the figure below.



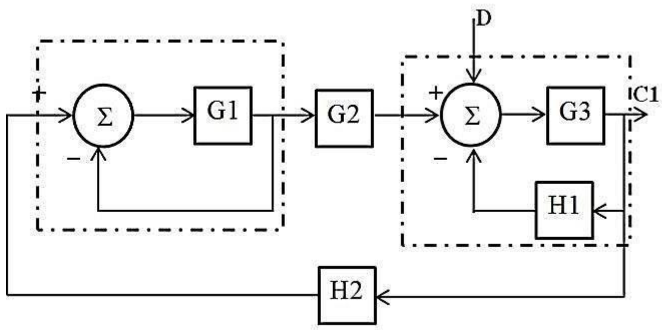
Step 2: Eliminating the feedback paths.



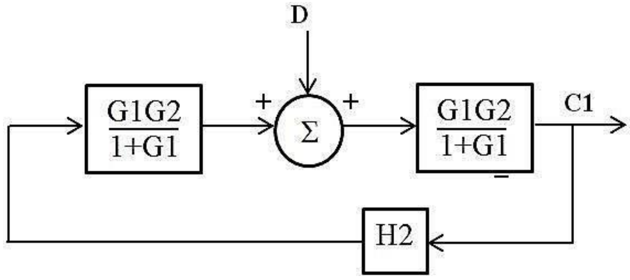
Step 3: The three forward path blocks and the feedback block is combined to give the transfer function.



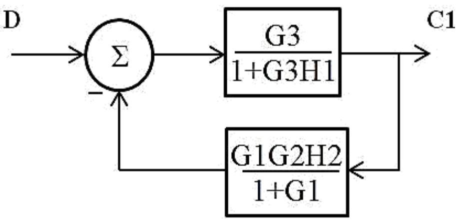
Step 4: Assuming R = 0 the block diagram in the question becomes



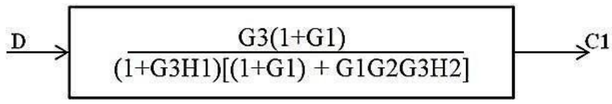
Step 5: The two feedback loops are eliminated.



Step 6: The block diagram can be redrawn as



Step 7: The block diagram can be reduced to give the transfer function as shown.



Step 8: When D = 0 output is C and is given below

$$C = \frac{RG1G2G3}{[(1+G1)(1+G3H1)+G1G2G3H2]}$$

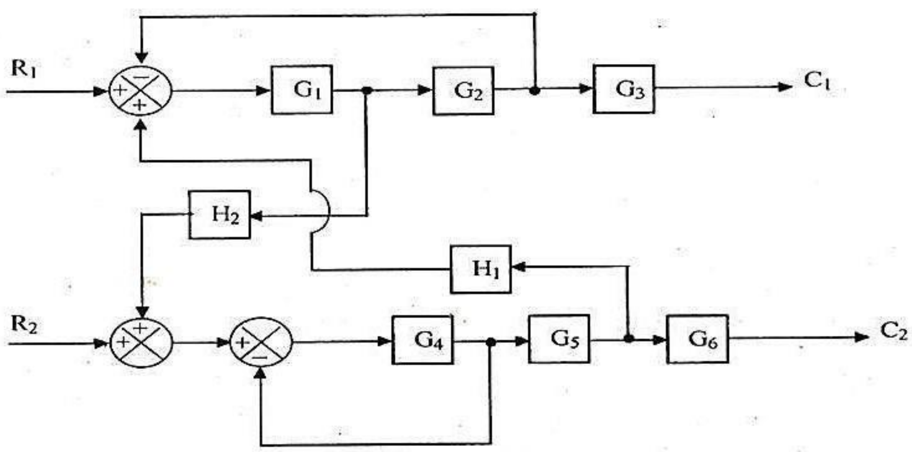
When R = 0 the output is C1 and is given by

$$C1 = \frac{DG3(1+G1)}{[(1+G1)(1+G3H1)+G1G2G3H2]}$$

When R and D are simultaneously present the output is O = C + C1

$$O = \frac{G3[RG1G2+D(1+G1)]}{[(1+G1)(1+G3H1)+G1G2G3H2]}$$

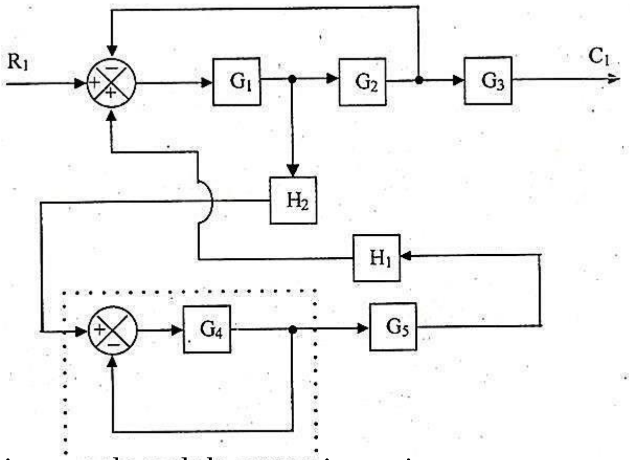
3.For the system represented by the block diagram shown in figure, Determine the transfer function C1/R1 and C2/R1.



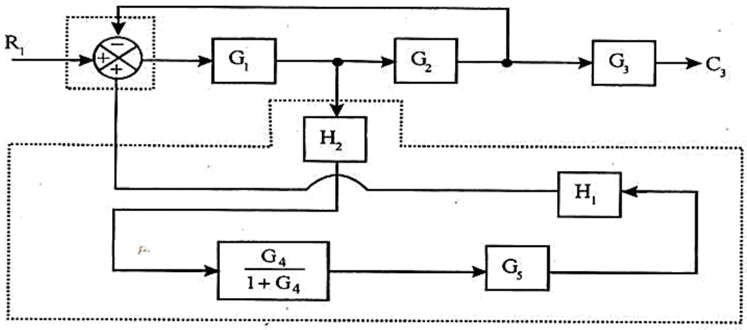
Solution:

Case 1: To find C_1/R_1. Consider R2 and C2 to be zero.

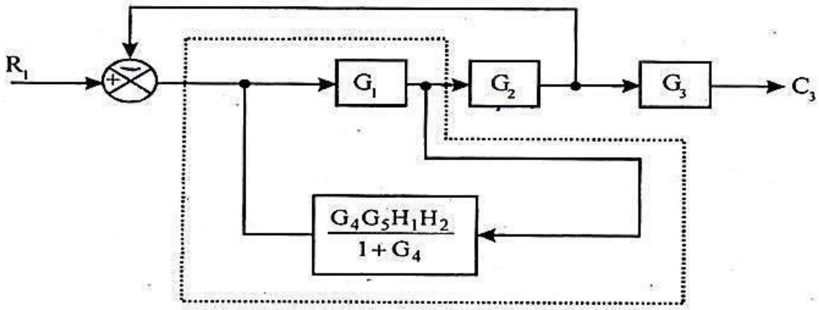
Step 1: Eliminate the feedback path.



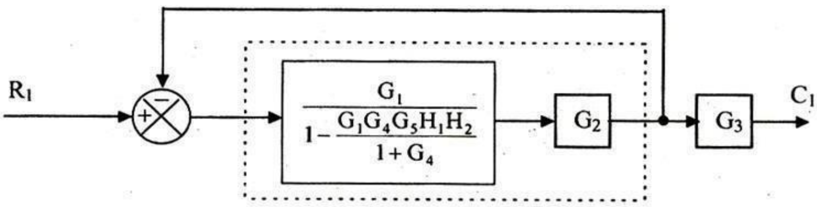
Step 2: Combining blocks in cascade and the summing point.



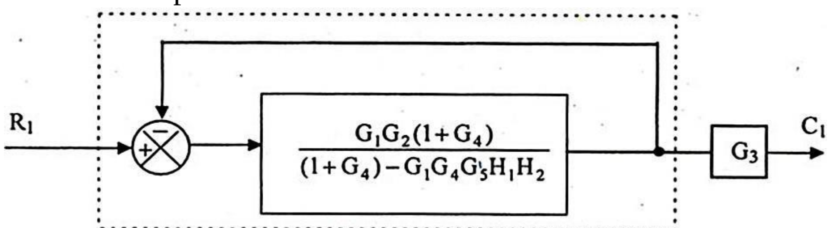
Step 3: Eliminate the feedback path.

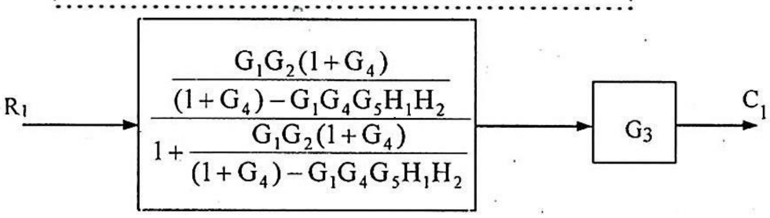


Step 4 : Combining the blocks in cascade

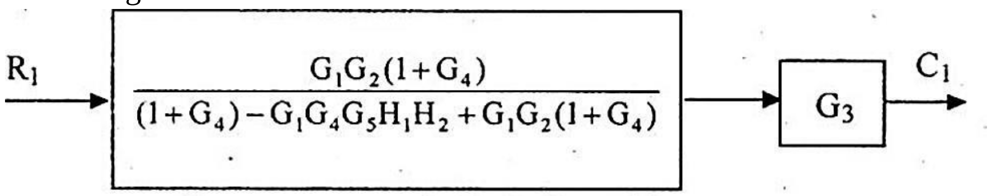


Step 5: Eliminate the feedback path.





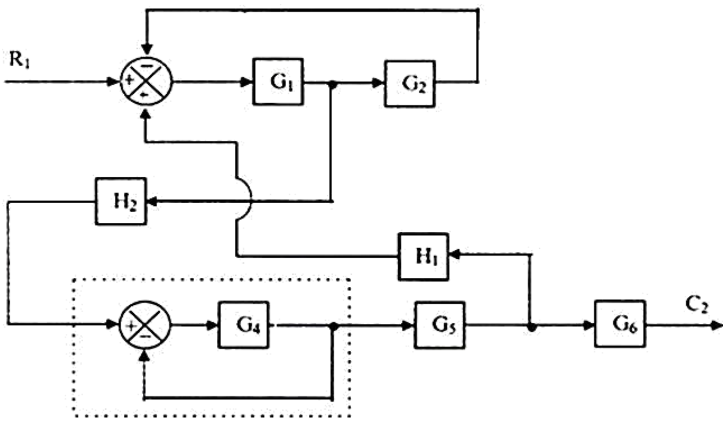
Step 6 : Combining the blocks in cascade



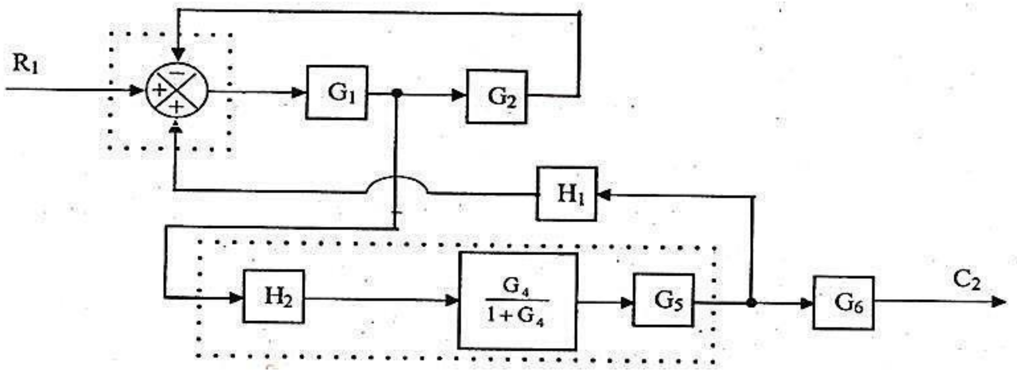
$$\frac{C_1}{R_1}=\frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)(1+G_4)-G_1G_4G_5H_1H_2}$$

Case 2: To find C_2/R_1. Consider R2 and C1 to be zero

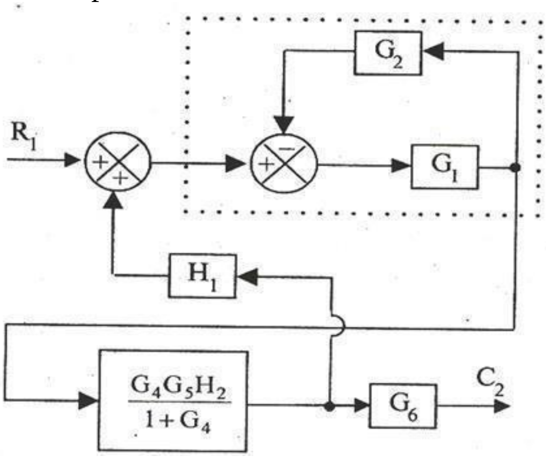
Step 1: Eliminate the feedback path.



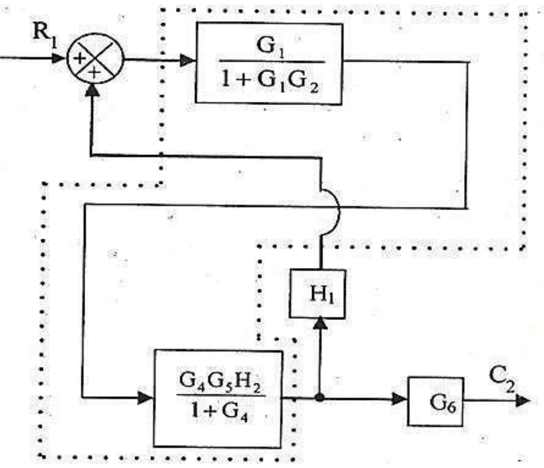
Step 2: Combining blocks in cascade and the summing point.



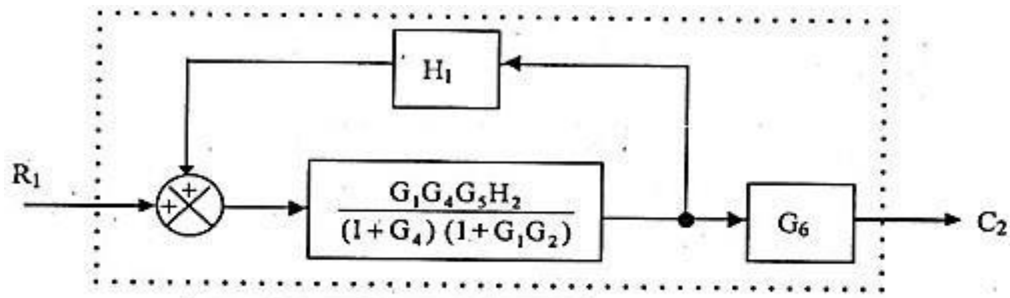
Step 3: Eliminate the feedback path.

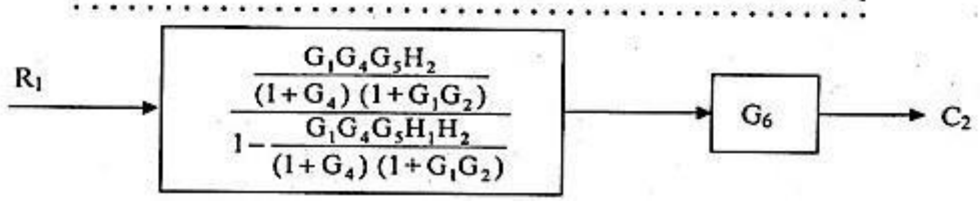


Step 4 : Combining the blocks in cascade

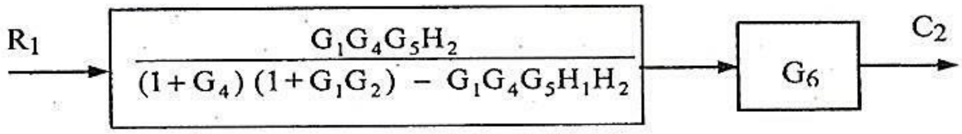


Step 5: Eliminate the feedback path.





Step 6 : Combining the blocks in cascade



$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)(1+G_1G_2) - G_1G_4G_5H_1H_2}$$

SIGNAL FLOW GRAPH

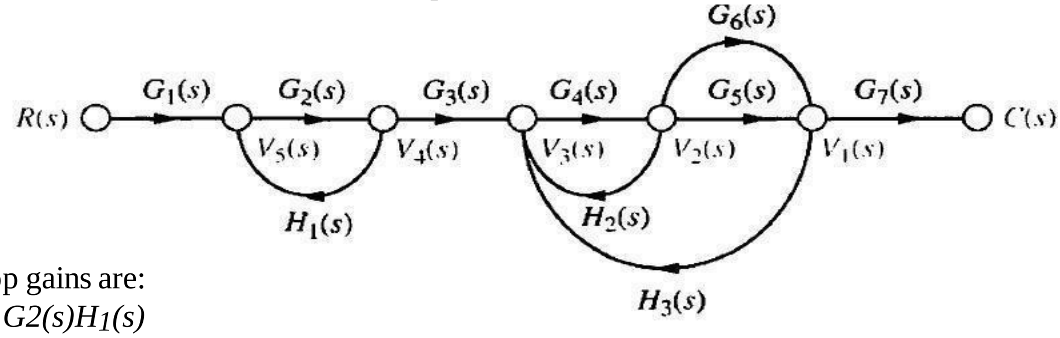
Signal-flow graphs represent transfer functions as lines, and signals as small circular nodes. Summing is implicit. Thus, the main advantage signal-flow graphs over block diagrams, is that they can be drawn more quickly, they are more compact, and they emphasize the state variables.

Signal-flow graphs are an alternative to block diagrams. Unlike block diagrams, which consist of blocks, signals, summing junctions, and pickoff points, a signal-flow graph consists only of *branches,* which represent systems, and *nodes,* which represent signals.

Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula. The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph *(Mason, 1953).* In general, it can be complicated to implement the formula without making mistakes.

Specifically, the existence of what we will later call nontouching loops increases the complexity of the formula. However, many systems do not have nontouching loops. For these systems, you may find Mason's rule easier to use than blockdiagram reduction. Mason's formula has several components that must be evaluated. First, we must be sure that the definitions of the components are well understood. Then we must exert care in evaluating the components with example then discuss the Mason"s Gain formula.

Loop gain. The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once. For example,



The loop gains are:

1. *G2(s)H1(s)*
2. *G4{s)H2{s)*
3. *G4(s)G5(s)H3(s)*
4. *G4(s)G6(s)H3(s)*

Forward-path gain: The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

1. *G1(s)G2(s)G3(s)G4(s)G5(s)G7(s)*
2. *G1(s)G2(s)G3(s)G4(s)G6(s)G7(s)*

Non touching loops : Loops that do not have any nodes in common. Loop G2(s)H1(s) does not touch loops G4(s)H2(s), G4(s)G5(s)H3(s) and G4(s)G6(s)H3(S)

Mason's Rule

The transfer function, T=*C(s)/R(s)*, of a system represented by a signal-flow graph is

$$T=\frac{1}{\Delta}\sum_{K} P_K\Delta_K$$

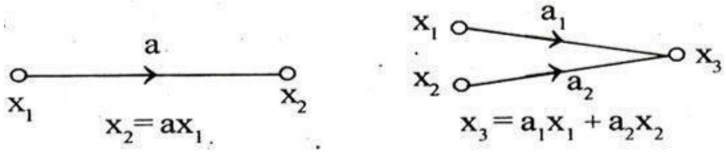
T= T(s) = Transfer function of the system
K= Number of forward path
P_k = Forward path gain of k^{th} forward path

Δ=1−(sum of all individual loopgains)
+(sum of gain products of all possible two nontouching loops)
−(sum of gain products of all possible threen ontouchingloops)+...

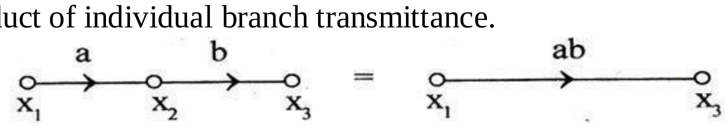
Δk = Δ for that part of the graph which is not touching k th forward path

Signal Flow Graph Algebra

Rule 1: Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.

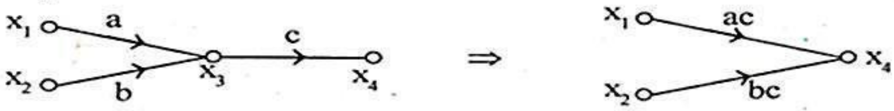


Rule 2: Cascade branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.

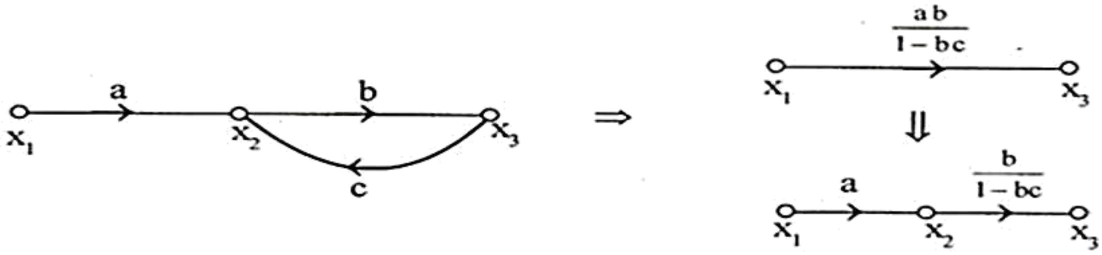


Rule 3: Parallel branch may be represented by single branch whose transmittance is the sum of individual branch transmittance.

Rule 4: A mixed node can be eliminated by multiplying the transmittance of outgoing branch (from the mixed node) to the transmittance of all incoming branches to the mixed node.



Rule 5: A loop may be eliminated by writing equations at the input and output node and rearranging the equation to find the ratio of output to input. This ratio gives the gain of resultant branch.



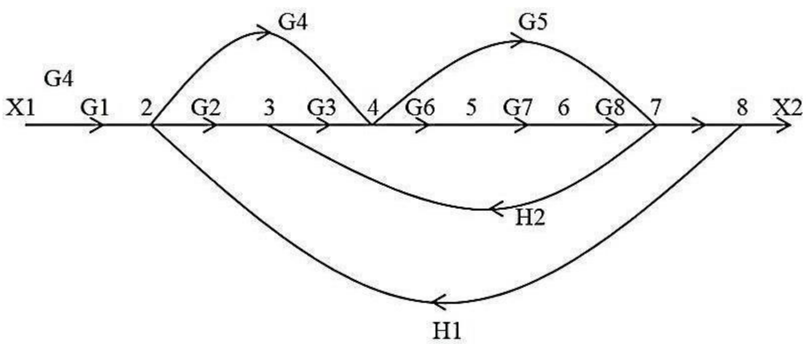
Steps to construct the Signal Flow Graph:

The Signal flow graph is constructed from its describing equations, or by direct reference to block diagram of the system. Each variable of the block diagram becomes a node and each block becomes a branch. The general procedure is

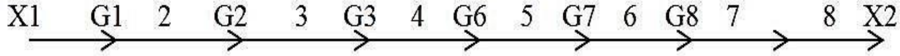
1. Arrange the input to output nodes from left to right
2. Connect the nodes by appropriate branches,
3. If the desired output node has outgoing branches, add a dummy node and a unity gain branch.
4. Rearrange the node and/or loops in the graph to achieve pictorial clarity.

Examples:

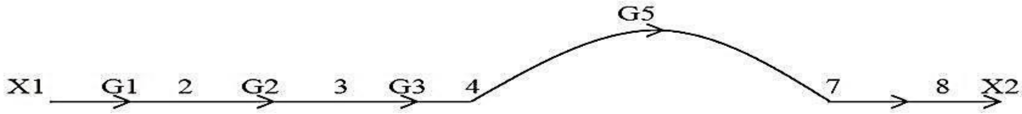
1. Determine the transfer function of the system using Mason's Gain formula.



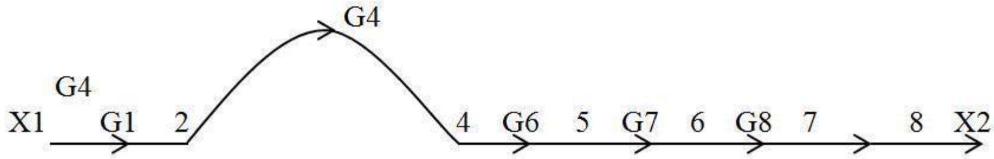
Step 1: There are four forward paths P1, P2, P3, P4; K = 4.



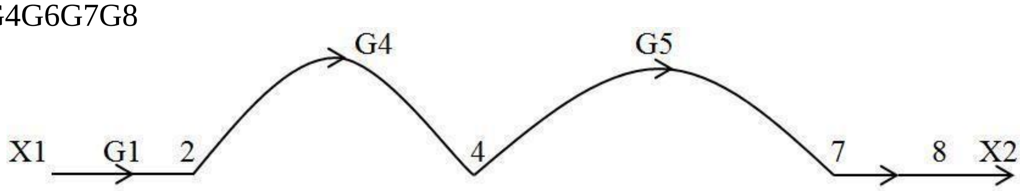
P1 = G1G2G3G6G7G8



P2 = G1G2G3G5G8

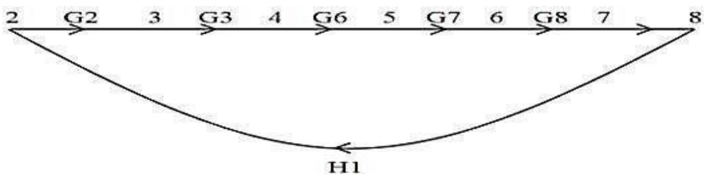


P3 = G1G4G6G7G8

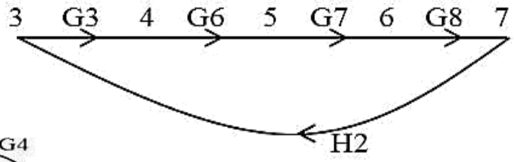


P4 = G1G4G5G8

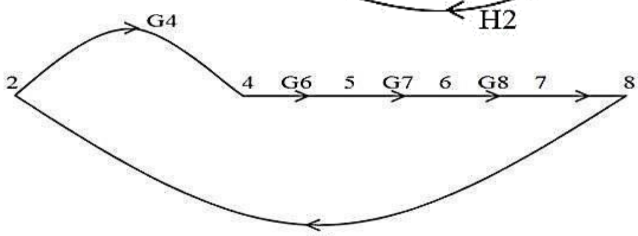
Step 2: There are 5 individual loops



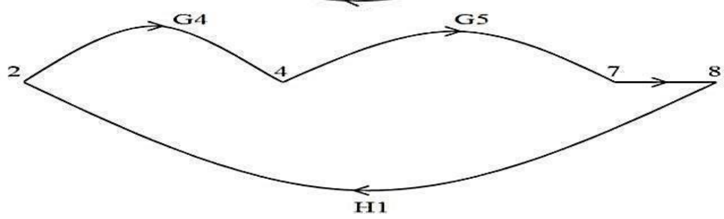
P11 = G2G3G6G7G8H1



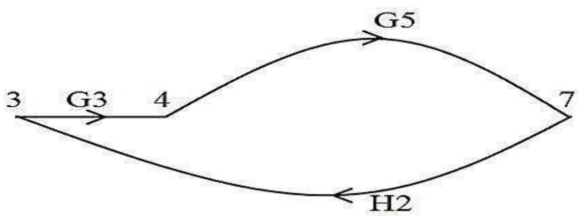
P21 = G3G6G7H2



P31 = G4G6G7G8H1



P41 = G4G5G8H1



P51 = G3G5H2

Step 4: There are no combination of two non-touching loops.

Step 5: Calculation of and ΔK

= 1 – (P11 + P21 +P31 + P41 + P51)

=1 – (G2G3G6G7G8H1 + G3G6H7H2 + G4G6G7G8H1 + G4G5G8H1 + G3G5H2)

There is no part of the graph touching with 1^{st} forward path; $\Delta 1 = 1$
There is no part of the graph touching with 2^{nd} forward path; $\Delta 1 = 1$
There is no part of the graph touching with 3^{rd} forward path; $\Delta 1 = 1$
There is no part of the graph touching with 4^{th} forward path; $\Delta 1 = 1$

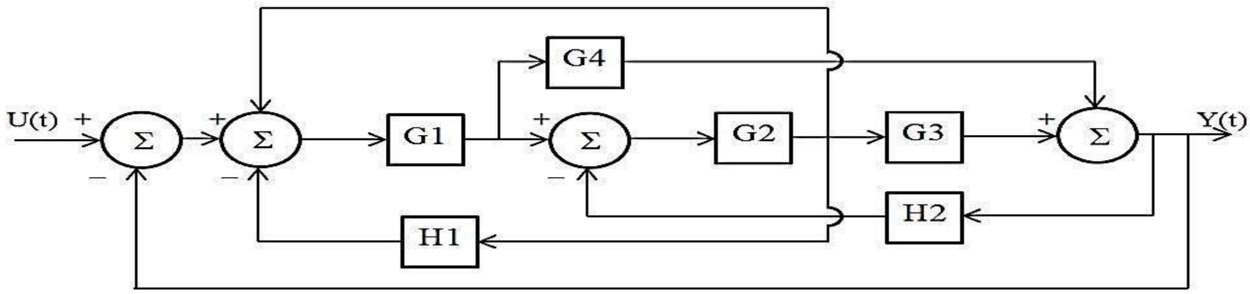
Step 6: Determination of transfer function.
By Masons gain formula the transfer function is given by

$$T = \frac{1}{\Delta} \sum_{K} PK\Delta K$$

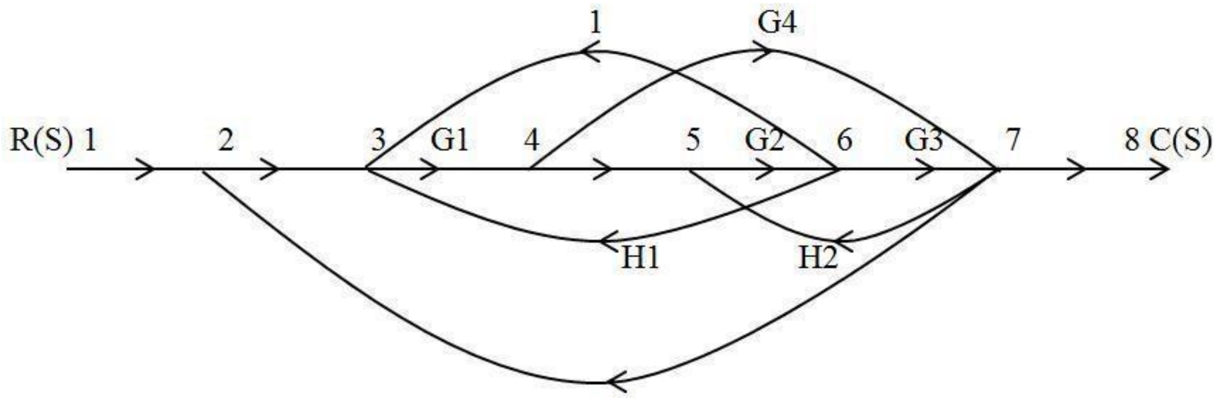
$$= 1/\Delta(P1\Delta1+P2\Delta2+P3\Delta3+P4\Delta4)$$

$$= \frac{G1G2G3G6G7G8+G1G2G3G5G8+G1G4G6G7G8+G1G4G5G8}{1- G2G3G6G7G8H1+G3G6H7+G4G6G7G8H1+G4G5G8H1+G3G5H2}$$

2.Use Masons gain formula to obtain C(S)/R(s) of the system shown below by using signal flow graph.

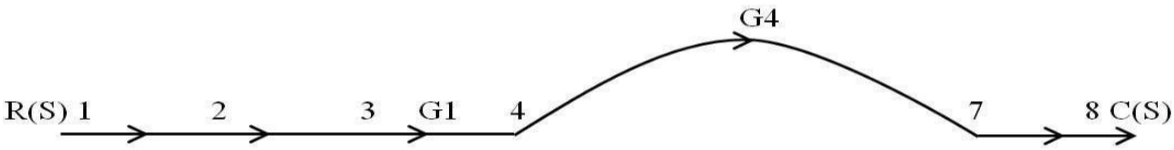


Step 1: The equivalent signal flow graph is given by



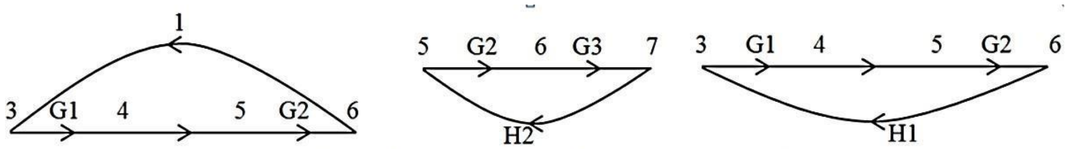
Step 2: There are two forward paths P1 and P2; K = 1 2

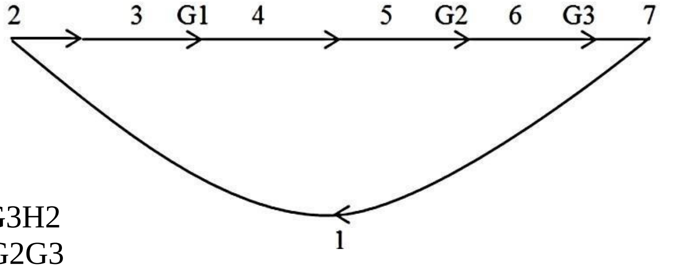
P1 = G1G2G3



P2 = G1G4

Step 3: There are 4 individual loops. Let the four loops be P11, P21, P31, P41





P11 = G1G2 P21 = G2G3H2
P31 = G1G2H1 P41 = G1G2G3

Step 3: Gain product of 2 non touching loops There are no 2 non touching loops. Step 4: Calculation of and ΔK

= 1 – (P11 + P21 + P31 + P41)
= 1 – G1G2 – G2G3H2 – G1G2H1 – G1G2G3

$\Delta 1 = 1 - 0 = 1$

$\Delta 2 = 1$

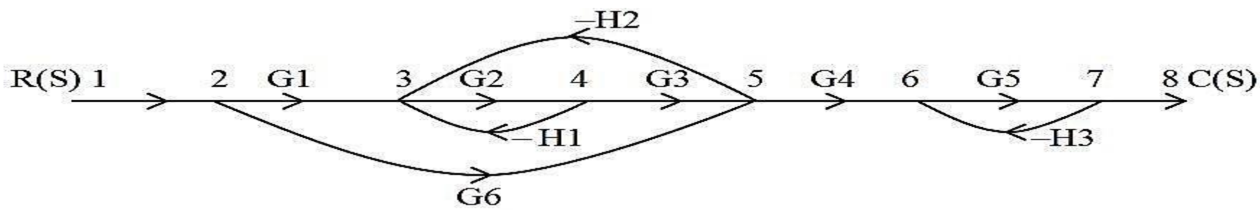
Step 5: Determination of transfer function

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

$= 1/\Delta(P1\Delta 1 + P2\Delta 2)$

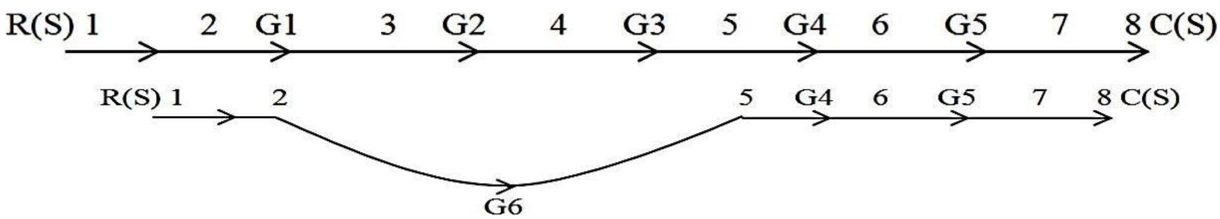
$$= \frac{G1G2G3 + G1G4}{1 - G1G2 - G2G3H2 - G1G2H1 - G1G2G3}$$

3.The signal flow graph for a feedback control system is shown in the figure. Determine the closed loop transfer function C(s)/R(s).



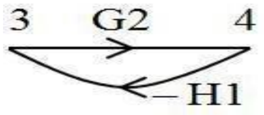
Step 1: Forward path gains
There are two forward paths K = 2

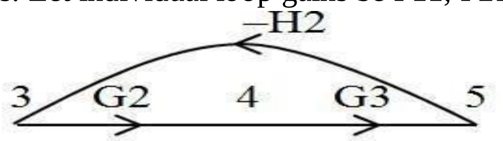
Let forward path gains be P1 and P2

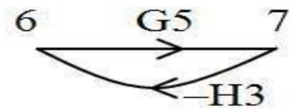


Gain forward path 1 = P1 = G1G2G3G4G5
Gain forward path 2 = P2 = G4G5G6
Step 2: Individual loop gain

There are three individual loops. Let individual loop gains be P11, P21, P31

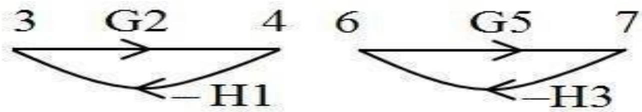






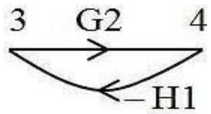
Loop gain of individual loop 1 = P11 = –G2H1
Loop gain of individual loop 2 = P12 = –G2G3H2
Loop gain of individual loop 3 = P31 = –G5H3

Step 3: Gain products of two non-touching loops
There are two combinations of two non-touching loops. Let the gain products of two non-touching loops be P12 and P22.



Gain product of 1^{st} combination of two non-touching loops P12 = P11P31 = (–G2H1)(–G5H3) = G2G5H1H3
Gain product of 2^{nd} combination of two non-touching loops
P22= P21P31= (–G2G3H2)(–G5H3)= G2G3G5G3H2H3

Step 4: Calculation of and ΔK
= 1– (P11 + P21 + P31) + (P12 + P22)
= 1– (–G2H1–G2G3H2–G5H3) + (G2G5H1H3 + G2G3G5G3H2H3)
= 1 + G2H1+G2G3H2+G5H3 + G2G5H1H3 + G2G3G5G3H2H3
Δ1 = 1, since there is no part of the graph which is not touching with first forward path The part of the graph which is not touching with the second forward path is shown below.



Δ2 = 1–P11 = 1 – (– G1H1) = 1+G2H1

Step 5: Transfer function T
By Mason"s gain formula the transfer function T is given by,

$$T = (1/\Delta).\sum P_K\Delta_K$$

$= (1/\Delta)(P1\Delta1+P2\Delta2)$ No of forward path is 2, K = 2

$$= \frac{G1G2G3G4G5+G4G5G6(1+G2H1)}{1+G2H1+G2G3H2+G5H3+G2G5H1H3+G2G3G5H2H3}$$

$$= \frac{G1G2G3G4G5+G4G5G6+G4G5G6G2H1)}{1+G2H1+G2G3H2+G5H3+G2G5H1H3+G2G3G5H2H3}$$

$$= \frac{G2G4G5[G1G3+G6/G2+G6H1]}{1+G2H1+G2G3H2+G5H3+G2G5H1H3+G2G3G5H2\ H3}$$

Servomechanism

A servo system mainly consists of three basic components - a controlled device, a output sensor, a feedback system. This is an automatic closed loop control system. Here, instead of controlling a device by applying the variable input signal, the device is controlled by a feedback signal generated by comparing output signal and reference input signal. When reference input signal or command signal is applied to the system, it is compared with output reference signal of the system produced by output sensor, and a third signal produced by a feedback system. This third signal acts as an input signal of controlled device.

This input signal to the device presents as long as there is a logical difference between reference input signal and the output signal of the system. After the device achieves its desired output, there will be no longer the logical difference between reference input signal and reference output signal of the system. Then, the third signal produced by comparing theses above said signals will not remain enough to operate the device further and to produce a further output of the system until the next reference input signal or command signal is applied to the system. Hence, the primary task of a servomechanism is to maintain the output of a system at the desired value in the presence of disturbances.

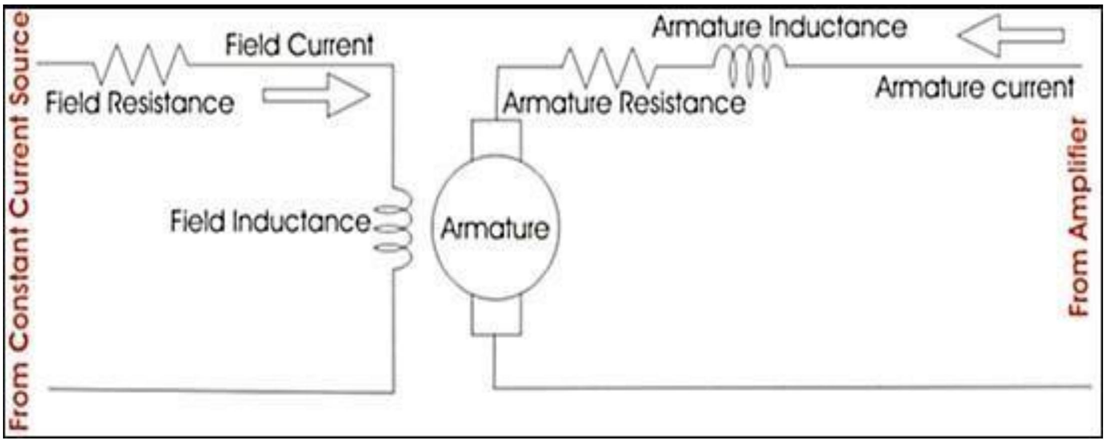
DC SERVO MOTOR

The motors which are utilized as DC servo motors generally have separate DC source for field winding and armature winding. The control can be achieved either by controlling the field current or armature current.

Armature Controlled DC Servo Motor

Theory:

The figure below shows the schematic diagram for an armature controlled DC servo motor. Here the armature is energized by amplified error signal and field is excited by a constant current source.



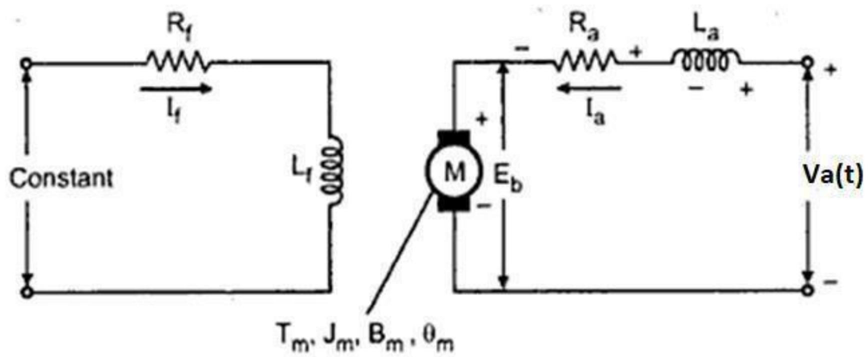
The field is operated at well beyond the knee point of magnetizing saturation curve. In this portion of the curve, for huge change in magnetizing current, there is very small change in mmf in the motor field. This makes the servo motor is less sensitive to change in field current. Actually for armature controlled DC servo motor, the motor should response to any change of field current.

Again, at saturation the field flux is maximum. The general torque equation of DC motor is, torque T aφI_a. Now if φ is large enough, for every little change in armature current I_a there will be a prominent changer in motor torque. That means servo motor becomes much sensitive to the armature current.

As the armature of DC motor is less inductive and more resistive, time constant of armature winding is small enough. This causes quick change of armature current due to sudden change in armature voltage. That is why dynamic response of armature controlled DC servo motor is much faster than that of field controlled DC servo motor.

The direction of rotation of the motor can easily be changed by reversing the polarity of the error signal.

Transfer Function:



Let

R_a = Armature resistance, Ω

L_a = Armature Inductance, H

I_a = Armature current, A

V_a = armature voltage, V

E_b = back emf, V, K_t = Torque constant, N-m/A

T = Torque developed by motor, N-m

θ = Angular displacement of shaft, rad

J = Moment of inertia of motor and load, Kg-m^2

B = Frictional coefficient of motor and load, N-m/(rad/sec)

K_b = Back emf constant, V/(rad/sec)

The differential equation of armature circuit is

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = V_a$$

Taking Laplace transform we get

$$L_a SI_a(s) + R_a I_a(s) + E_b(s) = V_a(s)$$

$$I_a(s)(L_a S + R_a) + E_b(s) = V_a(s)$$

$$I_a(s)(L_a S + R_a) = V_a(s) - E_b(s)$$

$$I_a(s) = \frac{V_a(s) - E_b(s)}{(L_a S + R_a)} \qquad [1]$$

Torque developed by motor is proportional to flux and current

$$T \alpha\, i_a \phi$$

$$T = K_T i_a$$

$$I_a(s) = \frac{T(s)}{K_T} \qquad [2]$$

According to Newton"s second law the Rotational mechanical differential equation is given by

$$J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

Taking Laplace transform

$$JS^2\theta(s) + BS\theta(s) = T(s) \qquad [3]$$

Also the back emf is proportional to the speed of shaft (Angular velocity)

$$e = K \frac{d\theta}{}$$

$$E_b(s) = K_b S\theta(s) \quad [4]$$

Combining equation [1] and [3], we get

$$\frac{T(s)}{K_T} = \frac{V_a(s) - E_b(s)}{(L_a S + R_a)}$$

$$T(s) = \frac{K_T V_a(s) - K_T E_b(s)}{(L_a S + R_a)}$$

Substituting [3] we get

$$JS^2\theta(s) + BS\theta(s) = \frac{K_T V_a(s) - K_T E_b(s)}{(L_a S + R_a)}$$

$$\theta(s)(JS^2 + BS)(L_a S + R_a) = K_T V_a(s) - K_T E_b(s)$$

$$K_T V_a(s) = [\theta(s)(JS^2 + BS)(L_a S + R_a)] + K_T E_b(s)$$

Substituting [4] we get

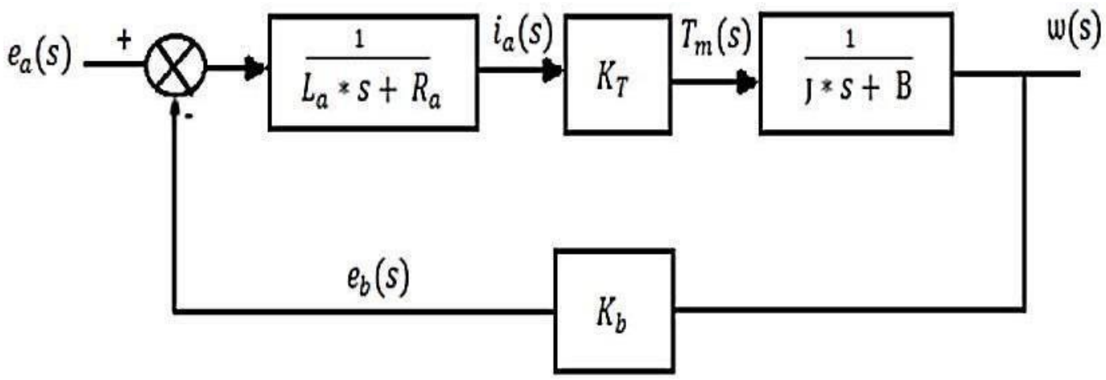
$$K_T V_a(s) = [\theta(s)(JS^2 + BS)(L_a S + R_a)] + K_T K_b S\theta(s)$$

$$K_T V_a(s) = \theta(s)[(JS^2 + BS)(L_a S + R_a) + K_T K_b S]$$

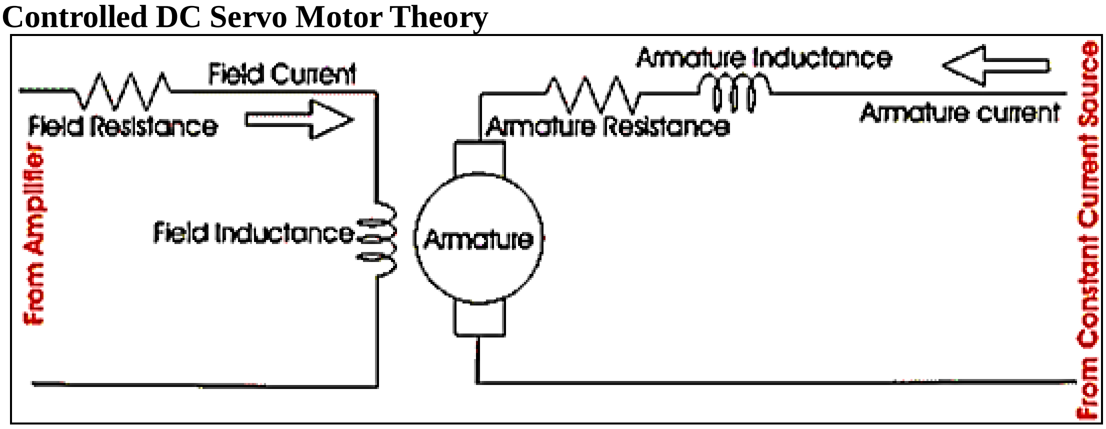
$$\frac{\theta(s)}{V(s)} = \frac{K_T}{(Js^2 + Bs)(L_a s + R_a) + K_T K_b s}$$

$$\frac{\theta(s)}{V(s)} = \frac{K_T}{(JL_a s^3) + s^2(L_a B + JR_a) + s(BR_a + K_T K_b)}$$

Block Diagram:



Field Controlled DC Servo Motor Theory



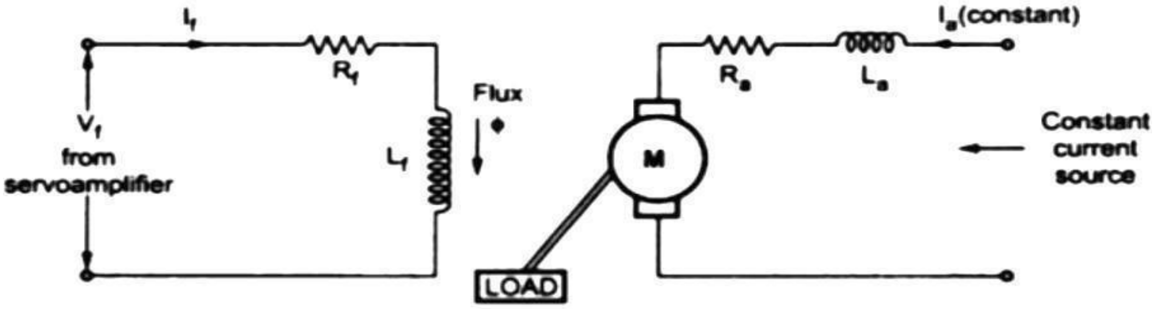
The figure illustrates the schematic diagram for a field controlled DC servo motor. In this arrangement the field of DC motor is excited be the amplified error signal and armature winding is energized by a constant current source . The field is controlled below the knee point of magnetizing saturation curve. At that portion of the curve the mmf linearly varies with excitation current. That means torque developed in the DC motor is directly proportional to the field current below the knee point of magnetizing saturation curve.

From general torque equation of DC motor it is found that, torque T aφI_a. Where, φ is field flux and I_a is armature current. But in field controlled DC servo motor, the armature is excited by constant current source, hence I_a is constant here. Hence, T a φ

As field of this DC servo motor is excited by amplified error signal, the torque of the motor i.e. rotation of the motor can be controlled by amplified error signal. If the constant armature current is large enough then, every little change in field current causes corresponding change in torque on the motor shaft. The direction of rotation can be changed by changing polarity of the field. The direction of rotation can also be altered by using split field DC motor, where the field winding is divided into two parts, one half of the winding is wound in clockwise direction and other half in wound in anticlockwise direction. The amplified error signal is fed to the junction point of these two halves of the field as shown in the figure. The magnetic field of both halves of the field winding opposes each other. During operation of the motor, magnetic field strength of one half dominates other depending upon the value of amplified error signal fed between these halves. Due to this, the DC servo motor rotates in a particular direction according to the amplified error signal voltage.

The main disadvantage of field control DC servo motors, is that the dynamic response to the error is slower because of longer time constant of inductive field circuit. The field is an electromagnet so it is basically a highly inductive circuit hence due to sudden change in error signal voltage, the current through the field will reach to its steady state value after certain period depending upon the time constant of the field circuit. That is why field control DC servo motor arrangement is mainly used in small servo motor applications. The main advantage of using field control scheme is that, as the motor is controlled by field - the controlling power requirement is much lower than rated power of the motor.

Transfer Function:



Let

R_f = Field resistance, Ω

L_f= Field inductance, H

I_f = Field current, A

V_f = Field voltage, V

T = Torque developed by motor, N-m

K_{tf} = Torque constant, N-m/A

J = Moment of inertia of rotor and load, Kg-m^2/rad

B = Frictional coefficient of rotor and load, N-m/(rad/sec)

$T \alpha \ i_a i_f$

$T = K_{TF} \, i_f$

$T(s) = K_{TF} I_f(s)$ [1]

The differential equation of armature circuit is

$$L_f \frac{di_f}{dt} + R_f i_f = e_f$$

$L_f S I_f(s) + R_f I_f(s) = E_f(s)$

$I_f(s)(L_f S + R_f) = E_f(s)$ [2]

According to Newton‟s second law the Rotational mechanical differential equation is given by

$$J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

Taking Laplace transform

$JS^2\theta(s) + BS\theta(s) = T(s)$

Substituting [1] we get,

$JS^2\theta(s) + BS\theta(s) = K_{TF} I_f(s)$

$$I_f(s) = \frac{\theta(s)(JS^2 + BS)}{K_{TF}}$$

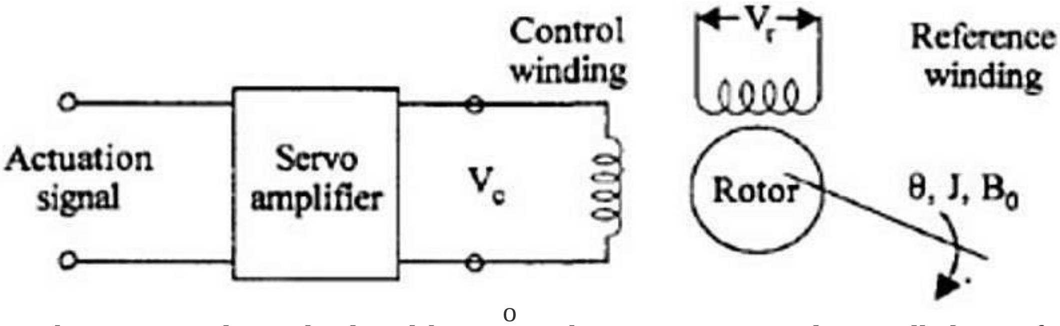
Substituting [4] in [2] we get,

$$\frac{\theta(s)(JS^2 + BS)}{K_{TF}}(L_f S + R_f) = E_f(s)$$

$$\frac{\theta(s)}{E_f(s)} = \frac{K_{TF}}{(JS^2 + BS)(L_f S + R_f)}$$

AC SERVOMOTOR

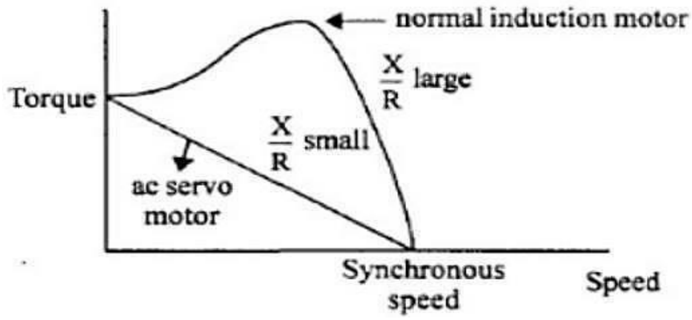
An AC servo motor is essentially a two phase induction motor with modified constructional features to suit servo applications.The schematic of a two phase or servo motor is shown



It has two windings displaced by 90° on the stator One winding, called as reference winding, is supplied with a constant sinusoidal voltage.The second winding, called control winding, is supplied with a variable control voltage which is displaced by -- 90° out of phase from the reference voltage.The major differences between the normal induction motor and an AC servo motor are

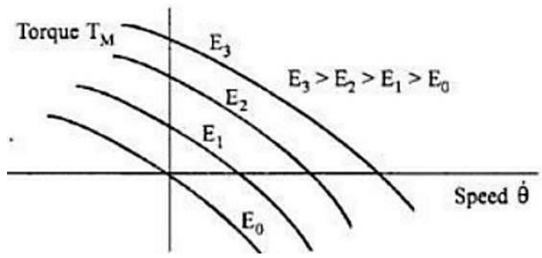
- The rotor winding of an ac servo motor has high resistance (R) compared to its inductive reactance (X) so that its X / *R* ratio is very low.
- For a normal induction motor, X / *R* ratio is high so that the maximum torque is obtained in normal operating region which is around 5% of slip.

  The torque speed characteristics of a normal induction motor and an ac servo motor are shown in fig

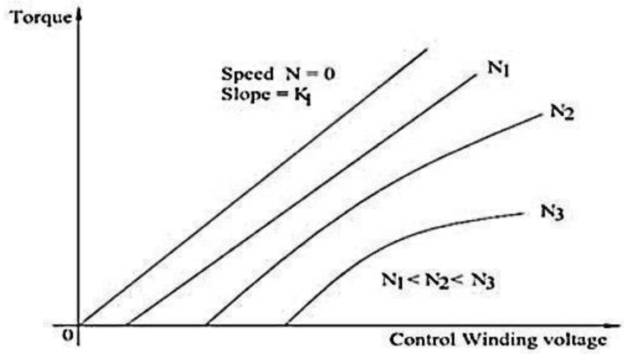


The Torque speed characteristic of a normal induction motor is highly nonlinear and has a positive slope for some portion of the curve. This is not desirable for control applications. as the positive slope makes the systems unstable. The torque speed characteristic of an ac servo motor is fairly linear and has negative slope throughout. The rotor construction is usually squirrel cage or drag cup type for an ac servo motor. The diameter is small compared to the length of the rotor which reduces inertia of the moving parts. Thus it has good accelerating characteristic and good dynamic response.

The supplies to the two windings of ac servo motor are not balanced as in the case of a normal induction motor. The control voltage varies both in magnitude and phase with respect to the constant reference vulture applied to the reference winding. The direction of rotation of the motor depends on the phase (± 90°) of the control voltage with respect to the reference voltage. For different rms values of control voltage the torque speed characteristics are shown in Fig.



The torque varies approximately linearly with respect to speed and also controls voltage. The torque speed characteristics can be linearized at the operating point and the transfer function of the motor can be obtained.



From the torque speed characteristics, we observe that even when Ec=0, the characteristics line runs through origin, which enables the stop of motor rapidly (decelerating torque). From torque-control voltage characteristics, we obtain that the high speed are nonlinear, so the AC servo motor is employed only for low speed.

With reference to the above characteristics, we assume that all lines are traight lines parallel to each other at rated input voltage ad are equally spaced for equal increments of input voltage.Under this assumption, the torque developed by the motor is,

$$T_m = K_1 e_C - K_2 \frac{d\theta}{dt}$$

From the mechanical system we get,

$$T_m = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

At equilibrium the motor torque is equal to load torque

$$K_1 e_C - K_2 \frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

$K_1 E_c(s) - K_2 S\theta(s) = JS^2\theta(s) + BS\theta(s)$

$K_1 Ec(s) = \theta(s)(JS^2 + BS + K_2 s)$

$$\frac{\theta(s)}{E_C(s)} = \frac{K_1}{(Js^2 + (B + K_2)s)}$$

$$\frac{\theta(s)}{E_C(s)} = \frac{\frac{K_1}{(B+K_2)}}{S\left(\frac{J}{(B+K_2)}S + 1\right)}$$

Let $K_m = \frac{K1}{(B+K_2)}$ be the motor gain constant

$r_m = \frac{J}{(B+K_2)}$ be the motor time constant

$$\frac{\theta(s)}{E_C(s)} = \frac{K_m}{S(r_m S + 1)}$$

SYNCHROS

The other names for synchros are Selsyn and autosyn. It is an electromagnetic transducer that produces an output voltage depending upon the angular displacement.it consists of two devices called Synchro Transmitter and Synchro Receiver. It is mostly used as an error detector in control system.

Synchro Transmitter:

It is similar to a Y connected 3-phase alternator. Stator winding are concentric coils displaced 120deg apart. Rotor is a salient pole type wound with concentric coils excited with single phase AC through slip rings. The Synchro transmitter acts as a transformer with single primary winding (Rotor) and there secondary winding displaced apart from each other.

The flux produced by the rotor is displaced along its axis and distributed sinusoidally in the air gaps depending upon its angular positions with rotor. Therefore the flux linked with the stator winding will induce an emf proportional to the cosine of the angle between the rotor and stator winding.

AC voltage applied across rotor $Vr(t) = A\sin\omega t$ Phase

voltage induced in stator coils S_1, S_2 and S_3 are

$Vs_1(t) = kA\sin\omega t \cos\theta$

$Vs_2(t) = kA\sin\omega t \cos(120 + \theta)$

$Vs_3(t) = kA\sin\omega t \cos(240 + \theta)$

Corresponding line voltage are

$$V_{L1} = V_{S2} - V_{S1}$$
$$V_{L1} = kA\sin\omega t[\cos(120+\theta) - \cos\theta]$$
$$V_{L1} = kA\sin\omega t(2\sin(60+\theta)\sin 60)$$
$$V_{L1} = kA\sin\omega t\sqrt{3}\sin(60+\theta)$$
$$V_{L2} = V_{S3} - V_{S2}$$
$$V_{L2} = kA\sin\omega t[\cos(240+\theta) - \cos(120+\theta)]$$
$$V_{L2} = kA\sin\omega t(\sin(180+\theta)\sin 60)$$
$$V_{L2} = \sqrt{3}kA\sin\omega t\sin(180+\theta)$$
$$V_{L3} = V_{S1} - V_{S3}$$
$$V_{L3} = kA\sin\omega t[\cos\theta - \cos(240+\theta)]$$
$$V_{L3} = -2kA\sin\omega t(\sin(120+\theta)\sin 120)$$
$$V_{L3} = \sqrt{3}kA\sin\omega t\sin(300+\theta)$$

When $\theta=0$; $V_{S1}(t) = kA\sin\omega t$ and $V_{L2} = 0$

The position at which V_{S1} is maximum and V_{L0} is zero is known as "electrical zero" or reference point of transmitter. The output of Synchro control transformer is the error signal which is proportional to the angular displacement between the two rotor of Synchro control transformer and Synchro transmitter.

Synchro control transformer:

The control transformer is similar in construction to a Synchro transmitter except the rotor is cylindrical in shape so that the air gap is uniform. Stator of both transmitter and transformer are identical and the output of the transmitter is given as input to the stator of Synchro transformer. A voltage will be induced in the rotor of control transformer by transformer action. This voltage is proportional to the cosine of the angle between the two rotors.

Therefore, $e(t) = k'A\sin\omega t\cos\phi$

Where ϕ- angular displacement between two rotors

When ϕ=90; e(t)=0, that is error voltage is zero.

The position is known as electrical zero or reference.

Let the initial position of rotor be 90 deg out of phase as in figure

$$e(t) = k'A\sin\omega t\cos 90 = 0$$

Let rotor transmitter is displaced by an angle θ and rotor of control transformer displaced by an angle α. Then the net displacement between the rotor is $(90+\theta-\alpha)$.

$$e(t) = k'A\sin\omega t\cos(90+\theta-\alpha) = k'A\sin\omega t\sin(\theta-\alpha)$$

For small angular displacement

$$e(t) = k'A(\theta-\alpha)\sin\omega t$$

Thus Synchro transmitter and control transformer acts as an error detector bygiving an error signal proportional to the angular difference between the transmitter and control transformer shaft position.

Input to the transmitter is a carrier signal error $(\theta-\alpha)$ acts as modulating signal error signal e(t) is a modulating signal .

FEEDBACK AND FEEDFORWARD CONTROL THEORY

In feedback system, when a disturbance enters the system, the process deviates, the error is sensed from the feedback. The control action is based on the error signal. The main disadvantage is that only after the disturbance enters the process, the controlled variable is deviated, then only the corrective action is taken.

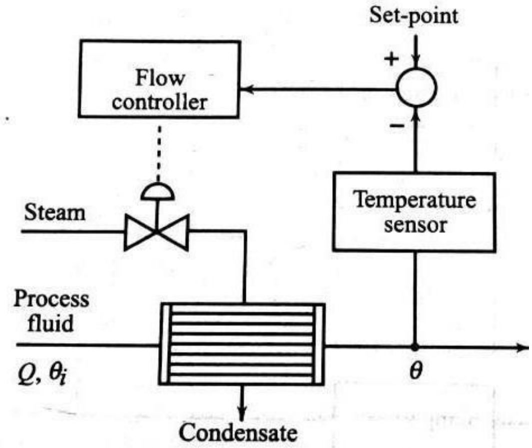


Fig: Feedback control

Whereas, in feed forward control system, the controller compensates before the disturbance affects the process. The efficiency of disturbance control depends on the ability to measure the disturbance. It estimates the effect of disturbance on the controlled variable, so that we can compensate for it.

For example, in a heat exchanger, the feedback control action depends on the sensed temperature. The input parameters to the plant are flow and temperature of the input fluid and the steam flow.

Any disturbance affecting the plant is sensed by the temperature sensor and then the control action is done by controlling the steam flow.

In feedforward control strategy the steam flow into the plant depends on the flow and temperature of the fluid. It is a kind of open loop control. The disturbance is anticipated prior to it affecting the plant. This control can minimize the transient error, with limited accuracy since it cannot cancel un-measurable disturbance.

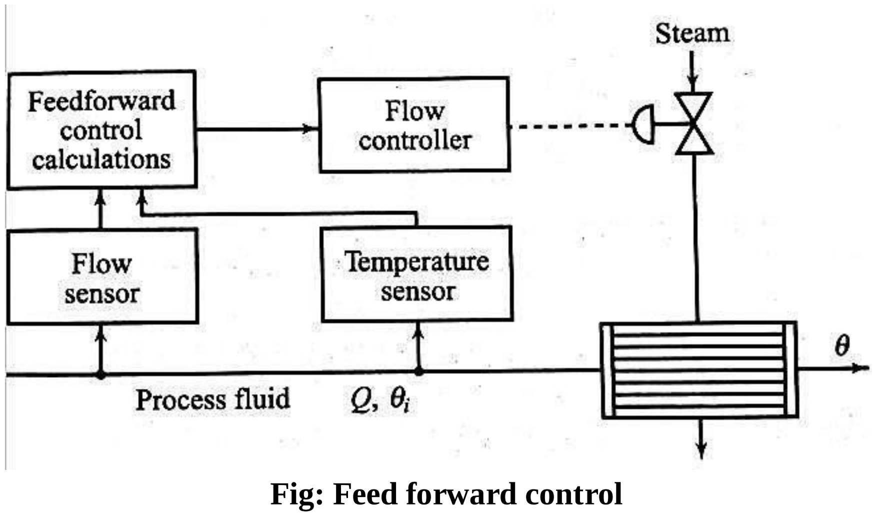


Fig: Feed forward control

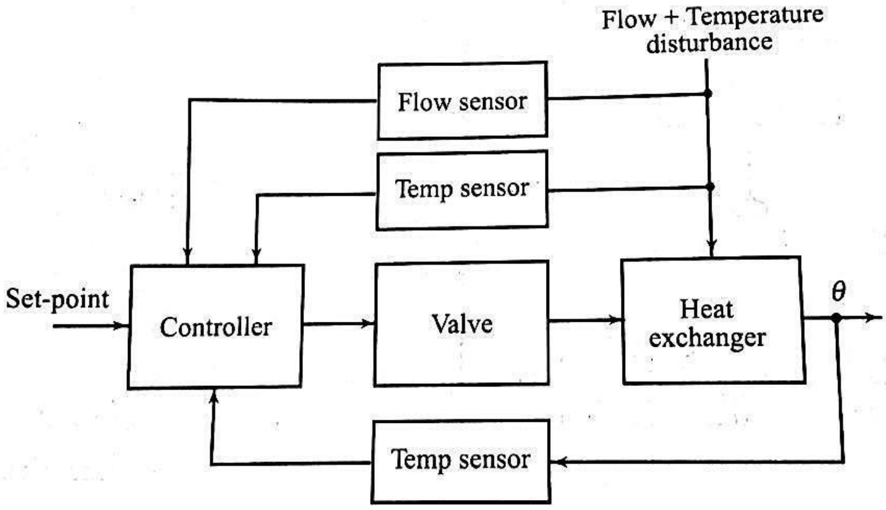


Fig: Feed forward control scheme

Another control scheme uses both Feed-forward and feedback control together, such that the system uses compensator and also provides the feedback control for unmeasurable disturbance.

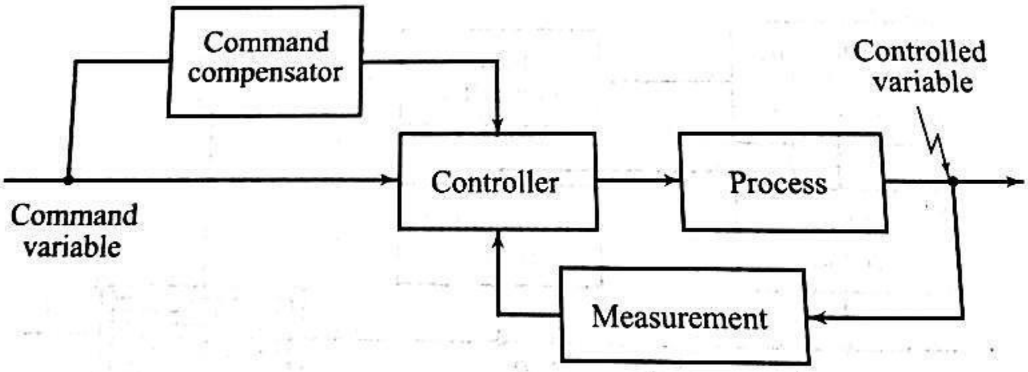


Fig: Combined Feed-forward and feedback control

MULTIVARIBLE CONTROL SCHEMES

Complex process and machines often have several variables (output) that we wish to control, and several manipulated input variables available to provide this control. Sometimes the control situation is simple; one input affects primarily one output and has only weak effect in the other outputs. In such situations, it is possible to ignore weak interactions (coupling) and design controllers under the assumption that one input affects only one output. Input-output paring to minimize the effects of interactions and application of SISO control schemes to obtain separate controllers for each input-output pair, results in an acceptable performance. This, in fact, amounts to considering the multivariable system as constituting of an appropriate number of separate SISO systems. Coupling effects are considered as disturbance to the separate control systems and may not cause significant degradation in their performance if the coupling is weak.

A multivariable system is said to have strong interaction (coupling) if one input affects more than one output appreciably. There are two approaches for the design of controllers for such system.

- Design a decoupling controller to cancel the interaction inherent in the system. Consider the resulting multivariable system as consisting of an appropriate number of SISO systems, and design a controller for each system.
- Design s single controller for the multivariable system, taking interacting into account.