UNIT III

DIGITAL SIGNALING FOR FADING CHANNELS

Structure of a wireless communication link, Principles of Offset-QPSK, p/4-DQPSK, Minimum Shift Keying, Gaussian Minimum Shift Keying, Error performance in fading channels, OFDM principle – Cyclic prefix, Windowing, PAPR.

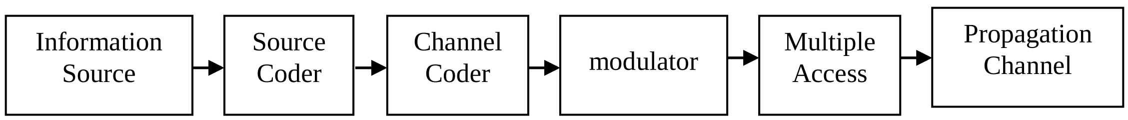
Structure of a Wireless Communication Link

BASIC TRANSCEIVER STRUCTURE

The goal of a wireless link is the transmission of information from an analog information source via an analog wireless propagation channel to an analog information sink.

Transmitter block

The simplified block diagram of transmitter is shown below.



Information source: The information source provides an analog source signal.

Source coder: The source coder reduces redundancy in the source signal. This reduces the amount of source data to be transmitted.

Channel coder: Channel coder adds redundancy in the form of a forward error correction code, in order to make it more resistant to errors introduced by the channel.

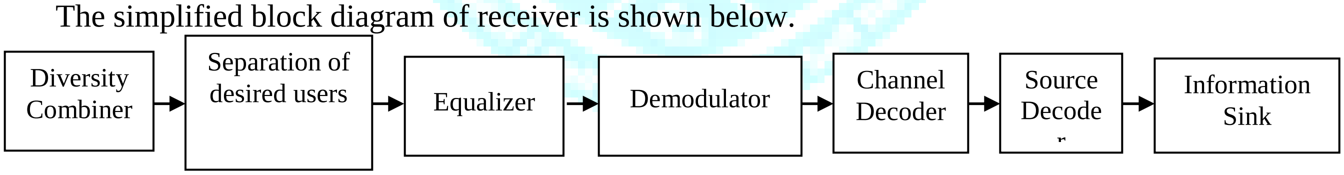
Modulator: The encoded data are then used as input to a modulator, which maps the data to output waveforms that can be transmitted.

Multiple Access: By transmitting the symbols on specific frequencies or at specific times, different users can be distinguished.

Propagation channel: The signal is then sent through the propagation channel. The channel attenuates and distorts the signal, and adds noise.

Receiver block

The simplified block diagram of receiver is shown below.



Diversity combiner: At the receiver, the signal is received by one or more antennas.

Separation of desired users: The different users are separated (e.g., by receiving signals only at a single frequency).

Equalizer: If the channel is delay dispersive, then an equalizer can be used to reverse that dispersion, and eliminate Intersymbol Interference.

Demodulator: Demodulator obtains *soft-decision* data from digitized baseband data, and hands them over to the channel decoder.

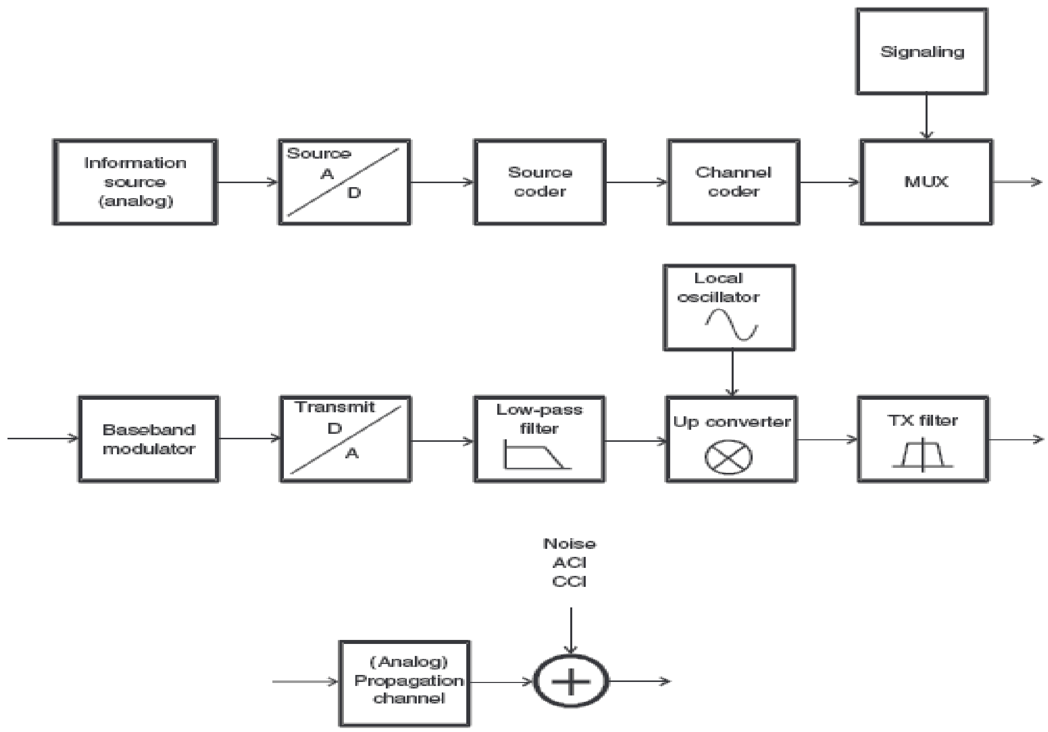
Channel decoder: Channel decoder eliminates the errors that are present in the resulting bitstream.

Source decoder: A source decoder finally maps this bitstream to an analog information stream that goes to the information sink (loudspeaker, TV monitor, etc.)

DETAILED BLOCK DIAGRAM OF A DIGITAL TX AND RX

Transmitter block

The block diagram of transmitter is shown below.



Information source: The information source provides an analog source signal and feeds it into the *source ADC*. This ADC converts the signal into a stream of digital data at a certain sampling rate and resolution.

Source coder: The source coder reduces redundancy in the source signal. This reduces the amount of source data to be transmitted, and thus the required transmission time and/or bandwidth

Channel coder: The channel coder adds redundancy in order to protect data against transmission errors. This increases the data rate that has to be transmitted.

Signaling: Signaling adds control information for the establishing and ending of connections, for associating information with the correct users, synchronization, etc.

Multiplexer: The multiplexer combines user data and signaling information, and combines the data from multiple users

Baseband modulator: The baseband modulator assigns the data bits to complex transmit symbols in the baseband. The output from the baseband modulator provides the transmit symbols in oversampled form, discrete in time and amplitude.

TX DAC: The TX Digital to Analog Converter (DAC) generates a pair of analog, discrete amplitude voltages corresponding to the real and imaginary part of the transmit symbols, respectively.

Analog low-pass filter: The analog low-pass filter in the TX eliminates the spectral components outside the desired transmission bandwidth.

TX Local Oscillator: The TX Local Oscillator (LO) provides an unmodulated sinusoidal signal, corresponding to one of the center frequencies of the considered system.

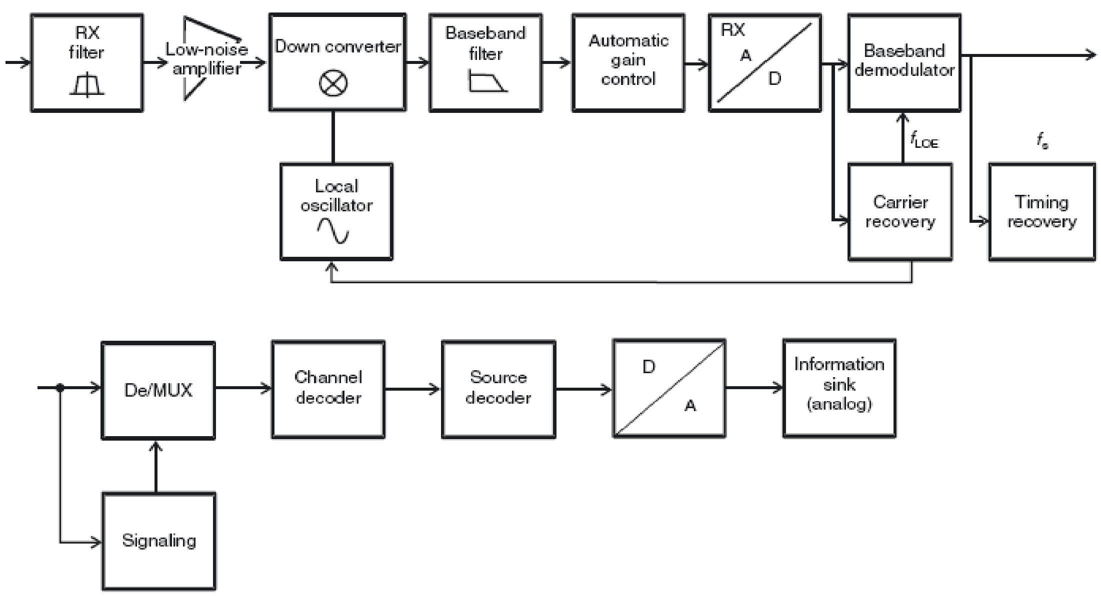
Upconverter: The upconverter converts the analog, filtered baseband signal to a passband signal by mixing it with the LO signal.

RF TX filter: Nonlinearities of mixers and amplifiers lead to creation of additional out-of-band emissions. The RF TX filter eliminates out-of-band emissions in the RF domain.

Propagation channel: The (analog) propagation channel attenuates the signal, and leads to delay and frequency dispersion. The environment adds noise and co-channel interference.

Receiver block

The block diagram of receiver is shown below.



RX filter: The RX filter performs a rough selection of the received band. The bandwidth of the filter corresponds to the total bandwidth assigned to a specific service.

Low-noise amplifier: The low-noise amplifier amplifies the signal, so that the noise added by later components of the RX chain has less effect on the Signal-to-Noise Ratio (SNR).

RX Local oscillator: The RX Local oscillator provides sinusoidal signals corresponding to possible signals at the TX LO. The frequency of the LO can be fine-tuned by a carrier recovery algorithm.

RX downconverter: The RX downconverter converts the received signal (in one or several steps) into baseband. In baseband, the signal is thus available as a complex analog signal.

RX low-pass filter: The RX low-pass filter provides a selection of desired frequency bands for one specific user. It eliminates adjacent channel interference as well as noise.

RX ADC: The RX ADC converts the analog signal into values that are discrete in time and amplitude.

Carrier recovery: The carrier recovery determines the frequency and phase of the carrier of the received signal, and uses it to adjust the RX LO.

Baseband demodulator: The baseband demodulator obtains *soft-decision* data from digitized baseband data, and hands them over to the decoder.

Symbol-timing recovery: The Symbol-timing recovery uses demodulated data to determine an estimate of the duration of symbols, and uses it to fine-tune sampling intervals.

Decoder: The decoder uses soft estimates from the demodulator to find the original (digital) source data.

Signaling recovery: Signaling recovery identifies the parts of the data that represent signaling information and controls the subsequent demultiplexer.

Demultiplexer: The demultiplexer separates the user data and signaling information and reverses possible time compression of the TX multiplexer.

Source decoder: The source decoder reconstructs the source signal from the rules of source coding. This is transferred to the data sink.

DIGITAL MODULATION TECHNIQUES

INTRODUCTION

Digital modulation is the mapping of data bits to signal waveforms that can be transmitted over an analog channel.

Advantages of digital modulation

Digital modulation offers many advantages over analog modulation. Some advantages include

- cost effective
- greater noise immunity
- robustness to channel impairments
- easier multiplexing of various forms of information (e.g., voice, data, and video)
- greater security

Factors That Influence the Choice of Digital Modulation

Several factors influence the choice of a digital modulation scheme.

The ***spectral efficiency*** of the modulation format should be as high as possible. This can best be achieved by a higher order modulation format. This allows the transmission of many data bits with each symbol.

Adjacent channel interference must be small. This entails that the power spectrum of the signal should show a strong roll-off outside the desired band. Furthermore, the signal must be filtered before transmission.

The ***sensitivity with respect to noise*** should be very small. This can be best achieved with a low-order modulation format, where (assuming equal average power) the difference between the waveforms of the alphabet is largest.

Robustness with respect to delay and Doppler dispersion should be as large as possible. Thus, the transmit signal should be filtered as little as possible, as filtering creates delay dispersion that makes the system more sensitive to channel-induced delay dispersion.

Waveforms should be ***easy to generate*** with hardware that is easy to produce and highly energy efficient.

A desirable modulation scheme

- provides low bit error rates at low received signal-to-noise ratios
- performs well in multipath and fading conditions
- occupies a minimum of bandwidth
- is cost-effective to implement.

The performance of a modulation scheme is often measured in terms of its

1. Power efficiency
2. Bandwidth efficiency.

Power efficiency: The ability of a modulation technique to preserve the fidelity of the digital message at low power levels is called **power efficiency**. It is expressed as the ratio of the *signal energy per bit* to *noise power spectral density* (E_b/N_0) required at the receiver input for a certain probability of error

Bandwidth efficiency: The ability of a modulation scheme to accommodate data within a limited bandwidth is called **bandwidth efficiency**. Bandwidth efficiency = R/B bps/Hz.

There is a upper bound on achievable bandwidth efficiency. Shannon's channel coding theorem states that for an arbitrarily small probability of error, the maximum possible bandwidth efficiency is limited by the noise in the channel, and is given by the channel capacity formula.

$$\eta_{B\ \max} = \frac{C}{B} = \log_2\left(1 + \frac{S}{N}\right)$$

QUADRATURE PHASE SHIFT KEYING (QPSK)

Quadrature phase shift keying (QPSK) has twice the bandwidth efficiency of BPSK, since 2 bits are transmitted in a single modulation symbol. The phase of the carrier takes on 1 of 4 values, such as 0, $\pi/2$, π, and $3\pi/2$,, where each value of phase corresponds to a unique pair of message bits.

The QPSK signal for this set of symbol states may be defined as

$$S_{QPSK}(t)=\sqrt{\frac{2E_s}{T_s}}\cos\left[2\pi f_c t+(i-1)\frac{\pi}{2}\right] \quad 0\le t\le T_s \quad i=1,2,3,4 \tag{1}$$

where T_s is the symbol duration and is equal to twice the bit period.

Using trigonometric identities, above equation can be rewritten for the interval $0 \le t \le T_s$

$$S_{QPSK}(t)=\sqrt{\frac{2E_s}{T_s}}\cos\left[(i-1)\frac{\pi}{2}\right]\cos(2\pi f_c t)-\sqrt{\frac{2E_s}{T_s}}\sin\left[(i-1)\frac{\pi}{2}\right]\sin(2\pi f_c t) \tag{2}$$

Consider basis functions

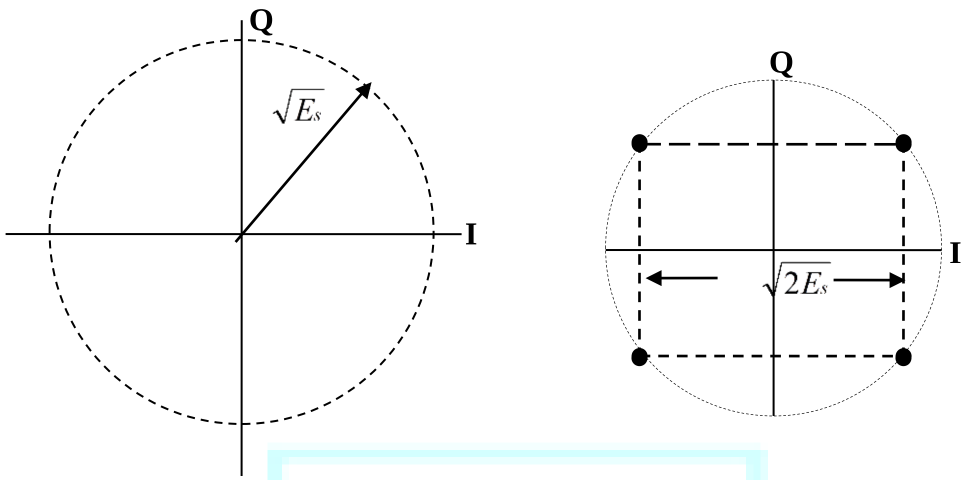
$$\phi_1(t)=\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$$

$$\phi_2(t)=\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$$

The 4 signals in the set can be expressed in terms of the basis signals as

$$S_{QPSK}(t)=\sqrt{E_s}\cos\left[(i-1)\frac{\pi}{2}\right]\phi_1(t)-\sqrt{E_s}\sin\left[(i-1)\frac{\pi}{2}\right]\phi_2(t) \tag{3}$$

QPSK Constellation



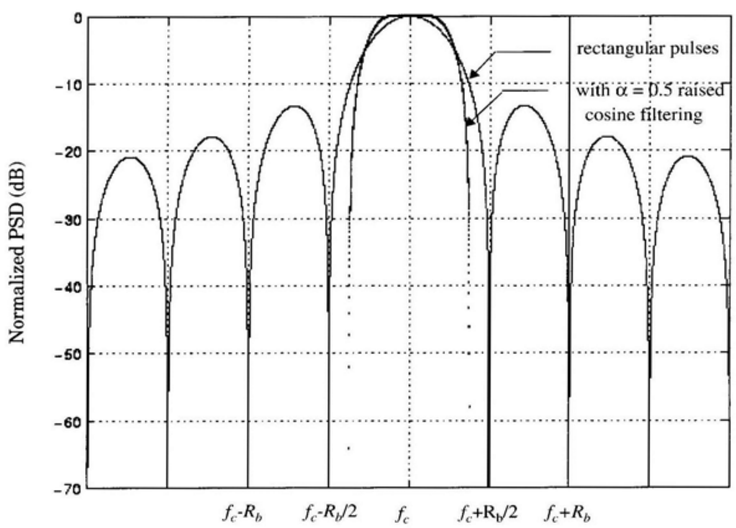
Carrier Phases 0,π/2,π,3π/2 Carrier Phases π/4,3π/4,5π/4,7π/4

The constellation diagram of QPSK

Since each symbol corresponds to two bits, then $E_s = 2E_b$. The average probability of bit error in the additive white Gaussian noise (AWGN) channel is given as

$$P_{e,\ QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad (4)$$

Spectrum and Bandwidth of QPSK Signals



The PSD of a QPSK signal using rectangular pulses is expressed as

$$\boldsymbol{P}_{\text{QPSK}}(f) = \frac{E_s}{2}\left[\left(\frac{\sin \pi(f - f_c)T_s}{\pi(f - f_c)T_s}\right)^2 + \left(\frac{\sin \pi(-f - f_c)T_s}{\pi(-f - f_c)T_s}\right)^2\right]$$

$$= E_b\left[\left(\frac{\sin 2\pi(f-f_c)T_b}{2\pi(f-f_c)T_b}\right)^2 + \left(\frac{\sin 2\pi(-f-f_c)T_b}{2\pi(-f-f_c)T_b}\right)^2\right]$$

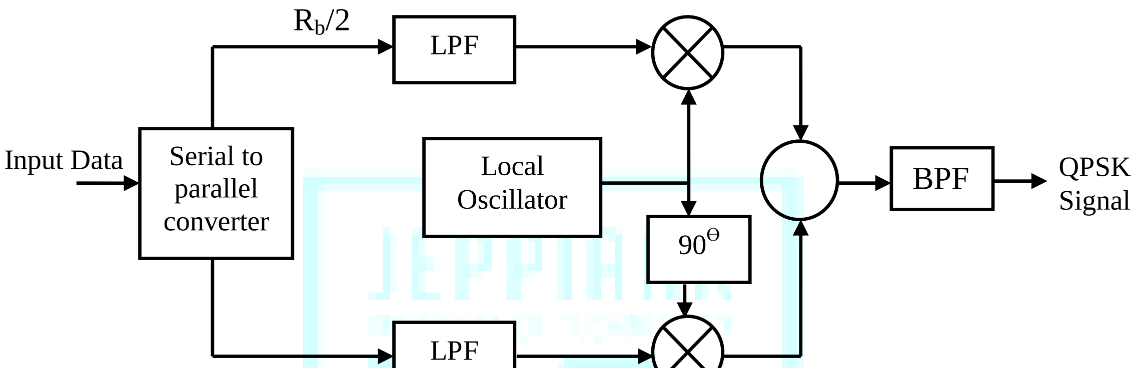
Where

$T_s = T_b/2 \quad \rightarrow$ symbol period

The null–to–null RF bandwidth is equal to the bit rate Rb, which is half that of a BPSK signal.

QPSK Transmitter

Figure shows a block diagram of a typical QPSK transmitter.



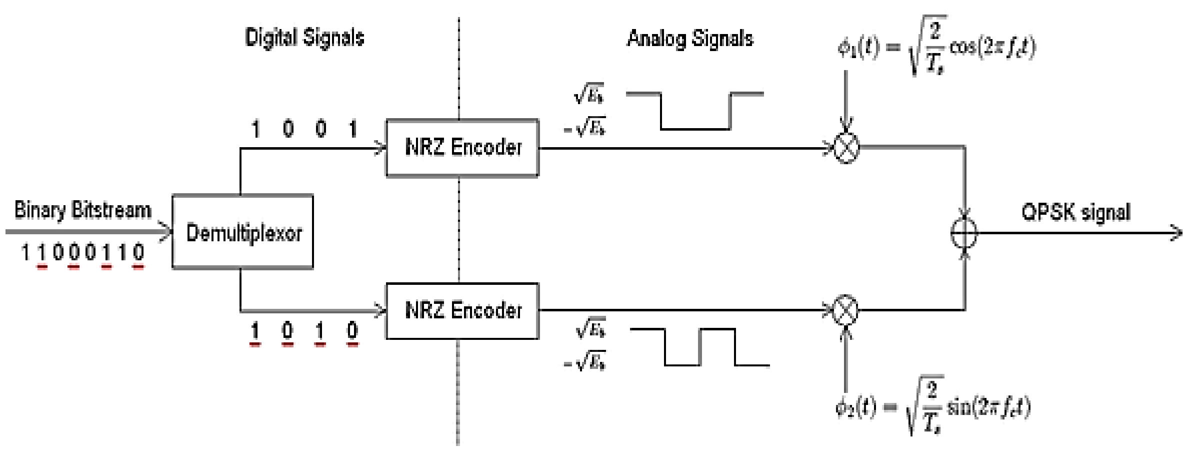
The unipolar binary message stream has bit rate R_b and is first converted into a polar form. This binary wave $m(t)$ is then split into two streams $m_I(t)$ and $m_Q(t)$ (in-phase and quadrature streams).

$$m_I(t) = p1_D(t) = \sum_{i=-\infty}^{\infty} b_{2i}\, g(t - iT_s) = b_{2i} * g(t)$$

$$m_Q(t) = p2_D(t) = \sum_{i=-\infty}^{\infty} b_{2i+1}\, g(t - iT_s) = b_{2i+1} * g(t)$$

(5)

Example:



Each stream has a bit rate of $R_s = R_b/2$. The binary stream $m_I(t)$ is called the even stream and $m_Q(t)$ is called the odd stream. The two binary sequences are separately modulated by two carriers $\phi_1(t)$ and $\phi_2(t)$. The two modulated signals are summed to produce a QPSK signal.

When interpreting QPSK as a *PAM*, the bandpass signal reads

$$S_{QPSK}(t) = \sqrt{\frac{E_b}{T_b}}\{p1_D(t)\cos(2\pi f_c t) - p2_D(t)\sin(2\pi f_c t)\} \qquad (6)$$

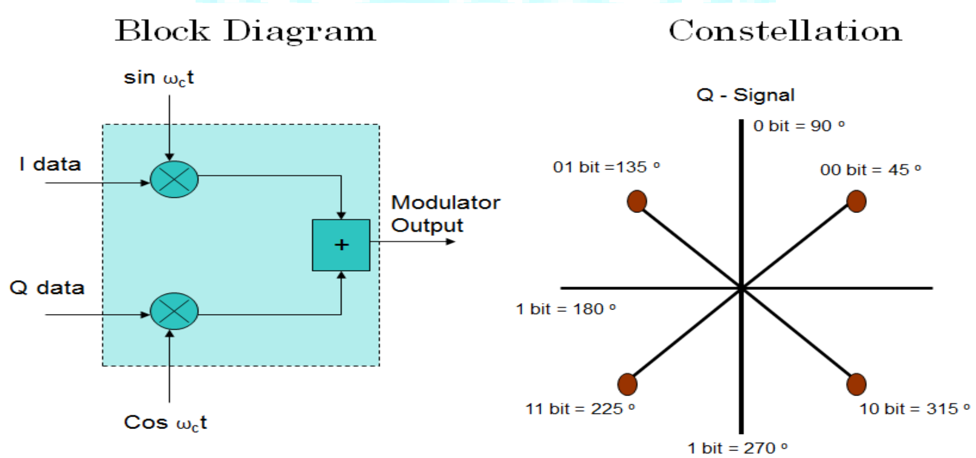
The baseband signal is

$$S_{QPSK}(t) = \sqrt{\frac{E_b}{T_b}}[p1_D(t) + jp2_D(t)] \qquad (7)$$

When interpreting QPSK as a *phase modulation*,

$$\phi(t) = \pi\left[\frac{1}{2}p2_D(t) - \frac{1}{4}p1_D(t)p2_D(t)\right] \qquad (8)$$

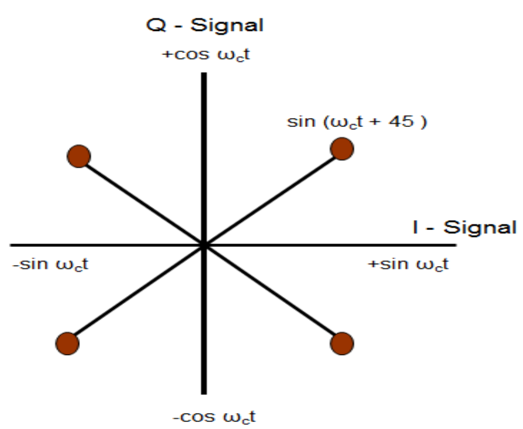
QPSK Modulator



Equations

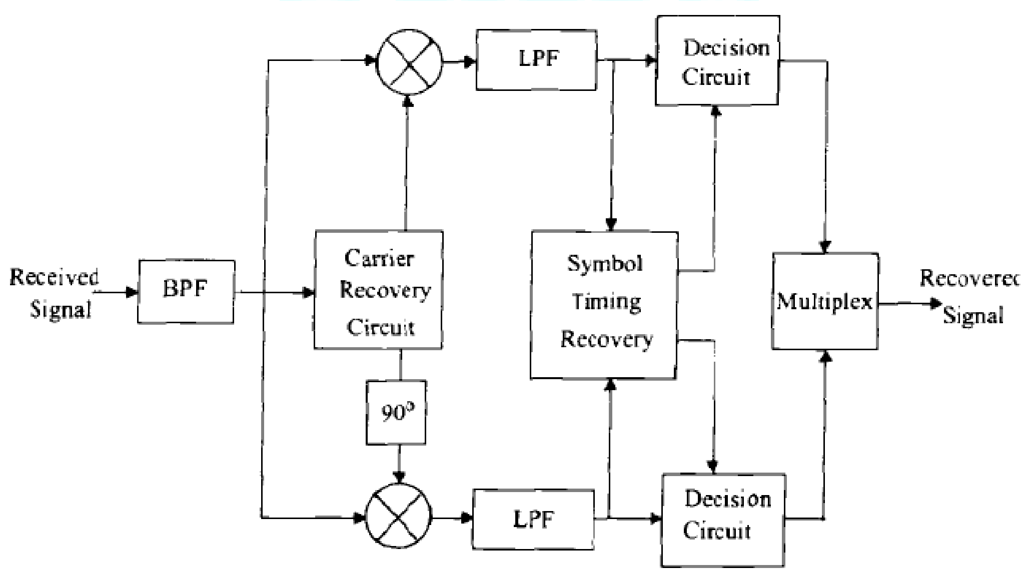
I Data	Q Data	I Mod O/P	Q Mod O/P	QPSK O/P	QPSK O/P Phase
0	0	$\sin \omega_c t$	$\cos \omega_c t$	$\sin \omega_c t + \cos \omega_c t = \sin (w_c t + 45)$	45°
0	1	$\sin \omega_c t$	$-\cos \omega_c t$	$\sin \omega_c t - \cos \omega_c t = \sin (\omega_c t + 135)$	135°
1	0	$- \sin \omega_c t$	$\cos \omega_c t$	$- \sin \omega_c t + \cos \omega_c t = \sin (\omega_c t - 45)$	315°
1	1	$- \sin \omega_c t$	$-\cos \omega_c t$	$- \sin \omega_c t - \cos \omega_c t = \sin (\omega_c t - 135)$	225°

Phasor Diagram

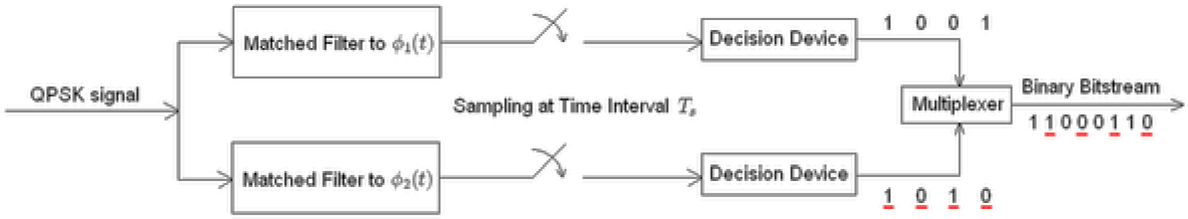


QPSK Receiver

Figure shows a block diagram of a coherent QPSK receiver.



The frontend bandpass filter removes the out-of-band noise and adjacent channel interference. The filtered output is split into two parts, and each part is coherently demodulated using the in-phase and quadrature carriers. The coherent carriers used for demodulation are recovered from the received signal using carrier recovery circuits. The outputs of the demodulators are passed through decision circuits which generate the in-phase and quadrature binary streams. The two components are then multiplexed to reproduce the original binary sequence.



Advantages & Disadvantages

Advantages:

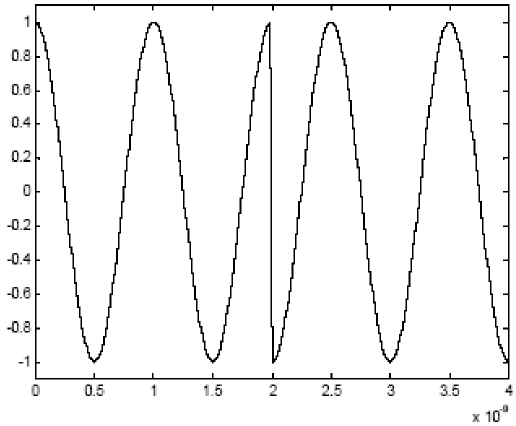
- Low error probability
  very good noise immunity
- For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK
- Because of reduced bandwidth, the information transmission rate of QPSK is higher and carrier power remains constant.
- Due to these advantages, the QPSK is used to achieve a very high bit rate.

Disadvantages:

- Very complex generation and detection
- Inter channel interference (ICI) is large due to side lobes
- QPSK signals are amplified by using linear amplifiers, which is less efficient
- Only suitable for rectangular data pulses
- QPSK phase changes by 90^{Θ} or 180^{Θ}. This creates an abrupt amplitude variations in the waveform.

OFFSET QPSK (OQPSK)

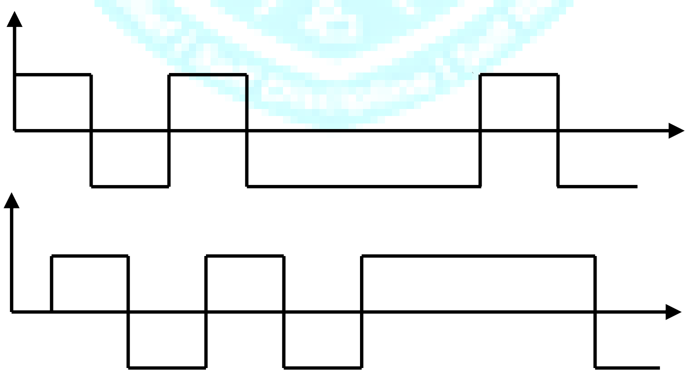
A modified form of QPSK, called offset QPSK (OQPSK) or staggered QPSK is less susceptible to these deleterious effects.
The occasional phase shift of π radians can cause the signal envelope to pass through zero for just in instant.
Any kind of hard limiting or nonlinear amplification of the zero-crossings brings back the filtered sidelobes. Since the fidelity of the signal at small voltage levels is lost in transmission.
OQPSK ensures there are fewer baseband signal transitions applied to the RF amplifier, helps to eliminate spectrum regrowth after amplification.
Example: First symbol (00) at 0º, and the next symbol (11) is at 180º. Notice the signal going through zero at 2 microseconds. This causes problems.



Using an offset approach: First symbol (00) at 0º, then an intermediate symbol at (10) at 90º, then the next full symbol (11) at 180º.
The intermediate symbol is used halfway through the symbol period.
It corresponds to allowing the first bit of the symbol to change halfway through the symbol period.
IS-95 uses OQPSK, so it is one of the major modulation schemes used.
In standard QPSK, the bit transition of even and odd bit streams (in-phase and quadrature components) occur at the same time. So phase transitions occur after $T_s = 2T_b$ sec. A maximum phase transition of 180° occurs if there is a change in the value of both $m_I(t)$and $m_Q(t)$.

The 180° phase transition in QPSK causes abrupt changes in the signal, resulting in large spectral side lobes. To prevent 180° phase changes in QPSK, **offset QPSK (OQPSK)** or **staggered QPSK (SQPSK)** is used.

In OQPSK, the in-phase and quadrature components of the standard QPSK are misaligned by T_b. Transitions for the in-phase component occur at nT_s while quadrature component transitions occur half a symbol duration (1-bit duration) later.



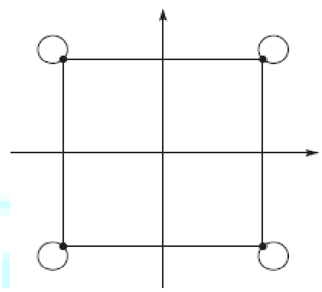
Misalignment of the in-phase and quadrature components prevents both components changing at the same time and thus prevents phase transitions of 180◦. This reduces the abrupt jumps in the modulated signal.

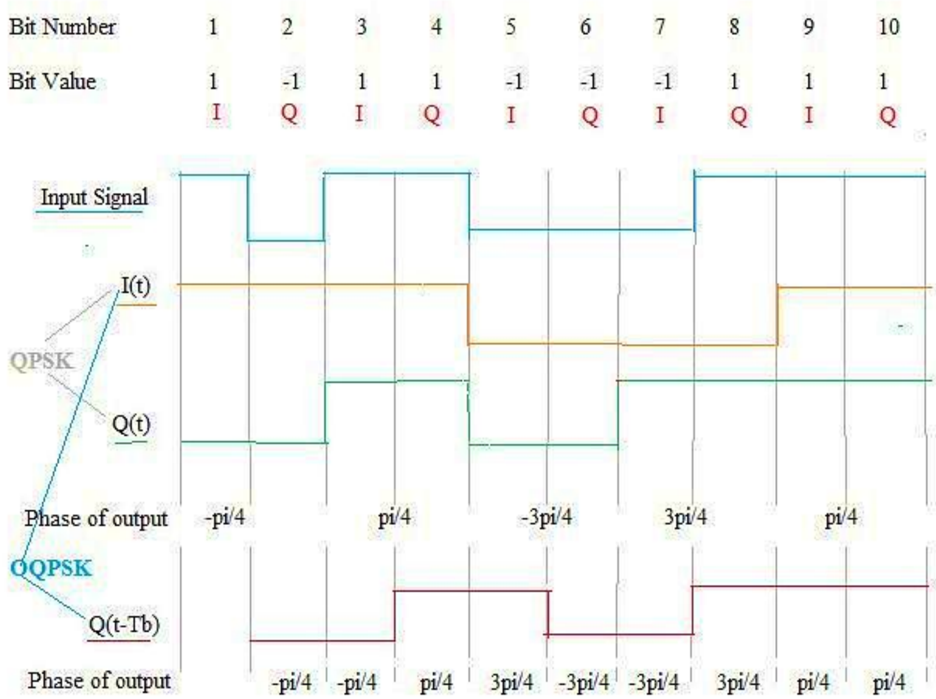
Thus, the transmit pulse streams are

$$m_I(t) = p1_D(t) = \sum_{i=-\infty}^{\infty} b_{2i}\, g(t - iT_s) = b_{2i} * g(t)$$

$$m_Q(t) = p2_D(t) = \sum_{i=-\infty}^{\infty} b_{2i+1}\, g\left(t - iT_s - \frac{T_s}{2}\right) = b_{2i+1} * g\left(t - \frac{T_s}{2}\right)$$

The phase transition diagram for OQPSK is shown in figure below. It is clear that there are no transitions passing through the origin of the coordinate system. Maximum phase transition is ±90°





QPSK and OQPSK Time Domain Waveforms

The spectrum is identical to that of a QPSK signal; hence both signals occupy the same bandwidth.

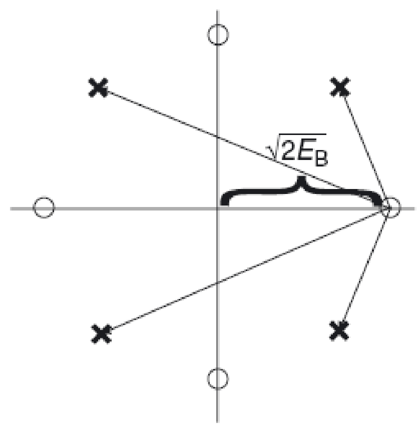
Advantages of offset QPSK, the first and the foremost is the maximum phase shift of the transmitted signal is limited to 90 degrees. That was my biggest fear in QPSK possibly there were several 180 degree phase shifts but since 180 degree phase shift have been removed in this case pulse shaping does not cause the offset QPSK signal envelop to go to zero. Hence nonlinear amplification does not generate high frequency side lobes, that’s a spectral occupancy is reduced allowing more efficient RF amplification so your receiver design also improves.

Π/4-DQPSK

The π/4 shifted QPSK modulation is a quadrature phase shift keying technique offers a compromise between OQPSK and QPSK in terms of the allowed maximum phase transitions.
It may be demodulated in a coherent or noncoherent fashion. Greatly simplifies receiver design.
In π/4 QPSK, the maximum phase change is limited to ± 135^o
In the presence of multipath spread and fading, π/4 QPSK performs better than OQPSK
Even though QPSK is a constant envelope format, it has amplitude dips at bit transitions. The duration of the dips is longer when non-rectangular basis pulses are used. Such variations of the signal envelope are undesirable, because they make the design of suitable amplifiers more difficult. This can be reduced by using ***π/*4-DQPSK** (*π*/4 differential quadrature-phase shift keying).

Principle of π/4-DQPSK

The principle of *π*/4-DQPSK can be understood from the signal space diagram of DQPSK.

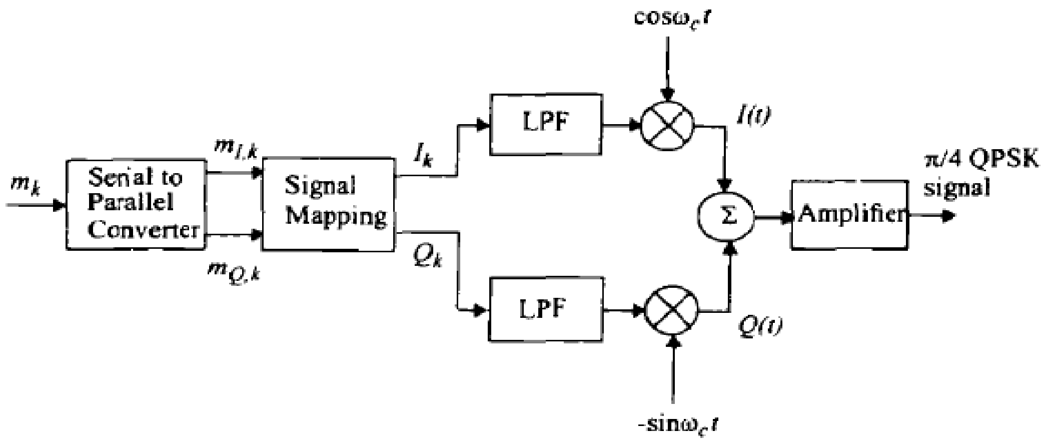


There exist *two* sets of signal constellations: (0, 90, 180, 270) and (45, 135, 225 315). All symbols with an even temporal index *i* are chosen from the first set, while all symbols with odd index are chosen from the second set.

In other words: whenever *t* is an integer multiple of the symbol duration, the transmit phase is increased by *π*/4, in addition to the change of phase due to the transmit symbol. Therefore, transitions between subsequent signal constellations can never pass through the origin

π/4 QPSK transmitter

A block diagram of *π*/4 QPSK transmitter is shown in Figure.



The input bit stream is partitioned by a serial-to-parallel (S/F) converter into two parallel data streams $m_{I,k}$ and $m_{Q,k}$. The kth in-phase and quadrature pulses, I_k and Q_k are produced at the output of the signal mapping.

$$I_k = \cos\theta_k$$
$$Q_k = \sin\theta_k$$

where

$$\theta_k = \theta_{k-1} + \phi_k$$

The phase shift ϕ_K is related to the input symbols $m_{I,k}$ and $m_{Q,k}$ according to Table.

Information bits $m_{I,k}$, $m_{Q,k}$	Phase shift ϕ_k
11	$\pi/4$
01	$3\pi/4$
00	$-3\pi/4$
10	$-\pi/4$

Just as in a QPSK modulator, the in-phase and quadrature bit streams I_k and Q_k are then separately modulated by two carriers which are in quadrature with one another, to produce the QPSK waveform given by

$$S_{\pi/4-DQPSK} = I(t)\cos\omega_c t - Q(t)\sin\omega_c t$$

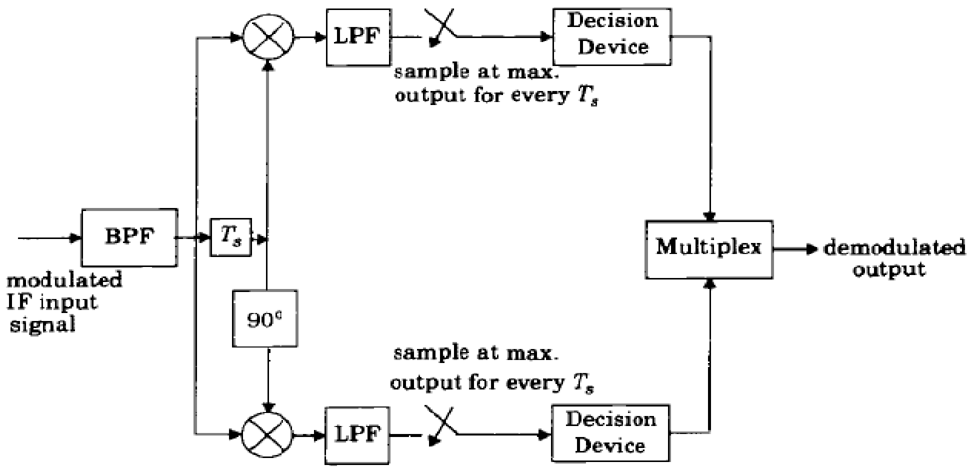
where

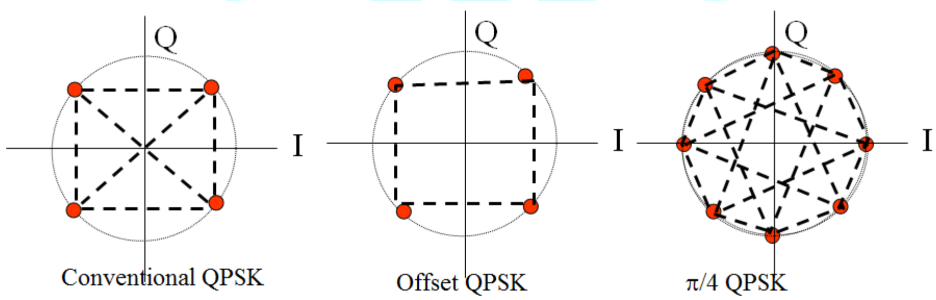
$$I(t) = \sum_{k=0}^{N-1} I_k g\left(t - kT_S - \frac{T_S}{2}\right)$$
$$Q(t) = \sum_{k=0}^{N-1} Q_k g\left(t - kT_S - \frac{T_S}{2}\right)$$

π/4 QPSK Detection (IF Differential Detector)

The IF differential detector shown in Figure below avoids the need for a local oscillator by using a delay line and two phase detectors.

The received signal is converted to IF and is bandpass filtered. The bandpass filter is designed to match the transmitted pulse shape, so that the carrier phase is preserved and noise power is minimized. The received IF signal is differentially decoded using a delay line and two mixers.





QPSK	OQPSK	pi/4 QPSK
phase changes of +/- 90 and +/-180 degrees	phase changes of +/- 90 exist	Maximum phase change of +/-45 and +/-135
Requirements of linear amplifier as non linear amplifier cause spectral regrowth because of abrupt +/- 180 degree transitions of the both bits change the phase at the same time.	less demands of linear amplifiers, efficient non linear amplifier can be employed and they do not cause much spectral regrowth, as one of the bits changes the phase at a time and occurs twice during the symbol period with half the intensity of QPSK	Phase transitions avoid zero crossing. This will remove design constaints on the amplifier, non linear amplifier can be employed

Null Bandwidth is 1.0 X Data rate	Same as QPSK	Same as QPSK
Bandwidth containing 90% of power is in 0.8 X Data rate	Same as QPSK	Same as QPSK
Power spectral density falls of as inverse second power of frequency	Same as QPSK	Same as QPSK
99% of power is contained in 1.0 X data rate	Same as QPSK	Same as QPSK
Amplitude variations are of the order of 30dB	Amplitude variation are of the order of 3 dB	-
Main lobe to side lobe suppression is poor	Same as QPSK	Same as QPSK
Main lobe width is 1.0 X data rate	Same as QPSK	Same as QPSK

PROBLEM:

Assume that $\Theta_0 = 0^{\Theta}$. The bit stream 0 0 1 0 1 1 is to be sent using $\Pi/4$ DQPSK. The leftmost bits are first applied to the transmitter. Determine the phase of Θ_K and the values of I_k, Q_k during transmission.

Given $\Theta_0 = 0^{\Theta}$

Formula used $I_k = \cos\theta_k$ $Q_k = \sin\theta_k$

$$\theta_k = \theta_{k-1} + \phi_k$$

We know that,

Information bits $m_{I,k}$, $m_{Q,k}$	Phase shift ϕ_k
11	$\pi/4$
01	$3\pi/4$
00	$-3\pi/4$
10	$-\pi/4$

The first two bits are 0 0, which implies that $\boldsymbol{\phi_1 = -3\pi/4}$

$$\theta_k = \theta_{k-1} + \phi_k$$

$$\boldsymbol{\Theta_1 = \Theta_0 + \phi_1 = -3\pi/4 \text{ (from table)}}$$

$$I_k = \cos\theta_k \qquad Q_k = \sin\theta_k$$

$I_1 = -0.707$ $Q_1 = -0.707$

Similarly,

The second two bits are 10, which implies that $\boldsymbol{\phi_2 = -\pi/4}$

$$\boldsymbol{\Theta_2 = \Theta_1 + \phi_2 = -3\pi/4 - \pi/4 = -\pi}$$

$I_2 = -1$ $Q_2 = 0$

The third two bits are 11, which implies that $\boldsymbol{\phi_3 = \pi/4}$

$$\boldsymbol{\Theta_3 = \Theta_2 + \phi_3 = -\pi + \pi/4 = -3\pi/4}$$

$I_3 = -0.707$ $Q_3 = -0.707$

MINIMUM SHIFT KEYING (MSK)

Minimum Shift Keying (MSK) is one of the most important modulation formats for wireless communications. However, it can be interpreted in different ways.

1. Minimum shift keying (MSK) is a special type of continuous phase FSK (CPFSK) where the peak frequency deviation, Δf is equal to 1/4 the bit rate, R_b.

$$\Delta f = \frac{1}{4}R_b = \frac{1}{4T_b}$$

MSK is continuous phase FSK with a modulation index of 0.5.

$$h_{\text{mod}} = 0.5$$

This implies that the phase changes by $\pm\pi/2$ during a 1-bit duration. A modulation index of 0.5 corresponds to the minimum frequency spacing that allows two FSK signals to be coherently orthogonal.

MSK is sometimes referred to as **fast FSK**, as the frequency spacing used is only half as much as that used in conventional noncoherent FSK.

2. An MSK signal is a special form of OQPSK where thebaseband rectangular pulses are replaced with half-sinusoidal pulses.for N-bit stream, MSK signal is

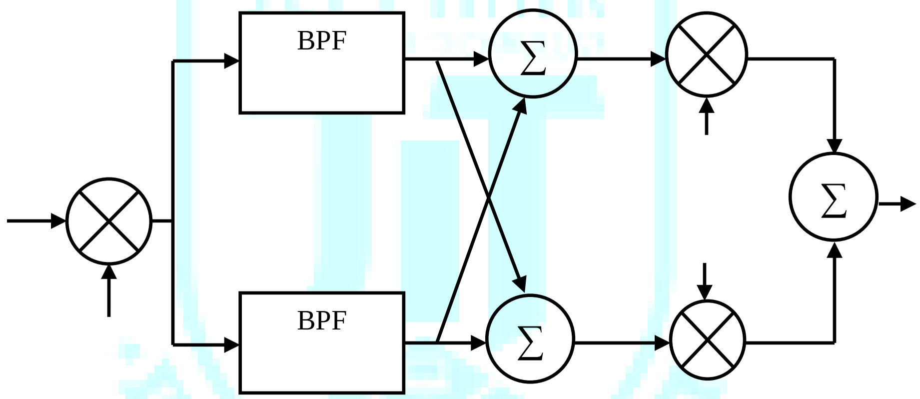
$$S_{MSK} = \sum_{i=0}^{N-1} m_I(t)g(t-2iT_b)\cos(2\pi f_c t) + \sum_{i=0}^{N-1} m_Q(t)g(t-2iT_b-T_b)\sin(2\pi f_c t)$$

where
$$g(t) = \begin{cases} \sin\left(\dfrac{\pi t}{2T_b}\right) & 0 \le t \le 2T_b \\ 0 & elsewhere \end{cases}$$

MSK Modulator

Multiplying a carrier signal $\cos(2\pi f_c t)$ with $\cos(\pi t/2T)$ produces two phase-coherent signals at $+1/4T$ and $-1/4T$. These two FSK signals are separated using two narrow bandpass filters and appropriately combined to form the in-phase and quadrature carrier components x(t) and y(t), respectively. These carriers are multiplied with the odd and even bit streams, $m_I(t)$ and $m_Q(t)$, to produce the MSK modulated signal.

MSK modulator is shown below.



Demodulation of Minimum Shift Keying

Different demodulator structures correspond to different interpretations:

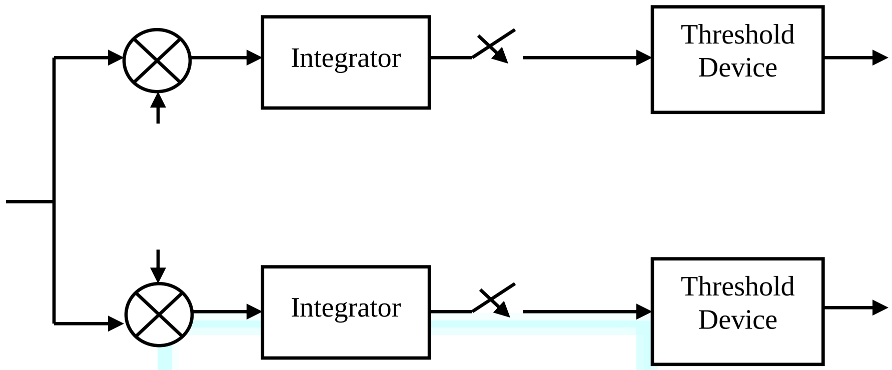
1. **Frequency discriminator detection:** Since MSK is a type of FSK, it is sufficient to check whether the instantaneous frequency is larger or smaller than the carrier frequency. The instantaneous frequency can be sampled in the middle of the bit, or it can be integrated over bit duration in order to reduce the effect of noise. This RX structure is simple, but suboptimum, since it does not exploit the continuity of the phase at bit transitions.

2. **Differential detection:** the phase of the signal changes by $+\pi/2$ or $-\pi/2$ over a 1-bit duration, depending on the bit that was transmitted. An RX thus just needs to determine the phases at times iT_b and $(i+1)T_b$, in order to make a decision. No differential encoding of the transmit signal is required.

3. **Matched filter reception:** it is well known that matched filter reception is optimum. This is true both when considering MSK as OQPSK, and when considering it as multipulse modulation. However, it has to be noted that MSK is a modulation format with memory.

Thus, bit-by-bit detection is suboptimum. Memory can be exploited, e.g., by a maximum-likelihood sequence estimation.

The block diagram of an MSK receiver is shown in Figure.



The received signal $S_{MSK}(t)$ (in the absence of noise and interference) is multiplied by the respective in-phase and quadrature carriers $x(t)$ and $y(t)$. The output of the multipliers are integrated over two bit periods and dumped to a decision circuit at the end of each two bit periods. Based on the level of the signal at the output of the integrator, the threshold detector decides whether the signal is a 0 or a 1. The output data streams correspond to $m_I(t)$ and $m_Q(t)$, which are offset combined to obtain the demodulated signal.

Properties of MSK

MSK is a spectrally efficient modulation scheme and is particularly attractive for use in mobile radio communication systems. It possesses properties such as

- constant envelope
- good spectral efficiency
- good BER performance
- self-synchronizing capability
- relatively narrow bandwidth
- coherent detection performance equivalent to that of QPSK

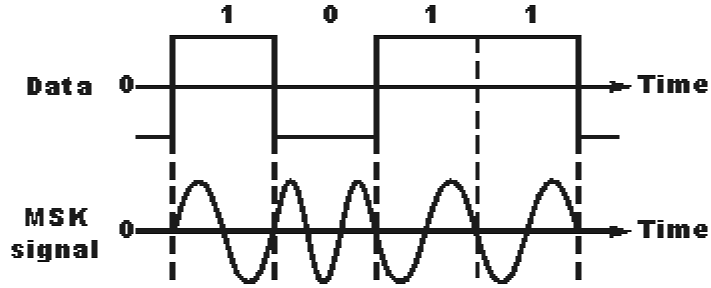
Specifications	QPSK modulation	MSK modulation
Full form	Quadrature Phase Shift Keying	Minimum Shift Keying
Maximum phase change	+/-90 , +/-180 degrees	phase change of +/-90 degree smoothly over course of a bit period
RF Amplifier requirement	requires linear amplifier. nonlinear amplifier if used will result into spectral regrowth due to +/-180 phase transition	phase change is linear and hence allows use of nonlinear amplifier
Null Bandwidth	equal to 1.0 times data rate	equal to 1.5 times data rate
Power	99% power is concentrated in 1.0(data rate)	99% power is concentrated in 1.2(data rate)
PSD(Power Spectral Density)	PSD falls off proportional to inverse second power of frequency	PSD falls off proportional to inverse fourth power of frequency
Amplitude variation	on the order of 30dB	very less
mainlobe to sidelobe suppression	Poor	is very high. Side lobes are much smaller compare to main lobe and hence filtering of MSK modulated signal is easier.
width of main lobe	1.0 times data rate	main lobe is wider than QPSK i.e. 1.5 times data rate
Definition	two BPSK in phase quadrature	two BPSK signals are orthogonal to one another in frequency quadrature

GAUSSIAN MINIMUM SHIFT KEYING (GMSK)

GMSK is a simple binary modulation scheme which may be viewed as a derivative of MSK.

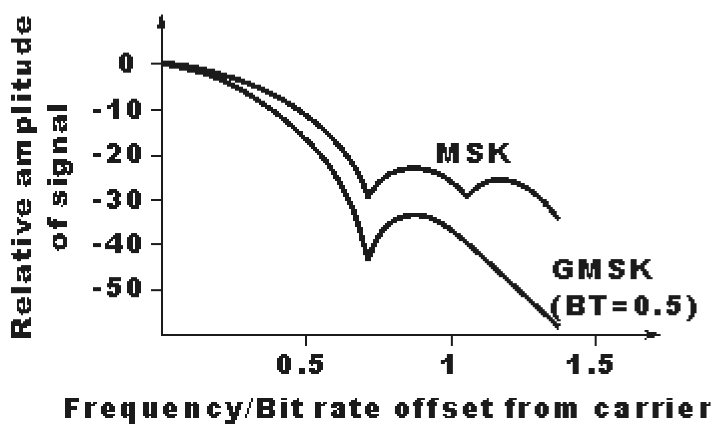
One of the problems with standard forms of PSK is that sidebands extend out from the carrier. To overcome this, MSK and its derivative GMSK can be used.

MSK and also GMSK modulation are what is known as a continuous phase scheme. Here there are no phase discontinuities because the frequency changes occur at the carrier zero crossing points. This arises as a result of the unique factor of MSK that the frequency difference between the logical one and logical zero states is always equal to half the data rate. This can be expressed in terms of the modulation index, and it is always equal to 0.5.



A plot of the spectrum of an MSK signal shows sidebands extending well beyond a bandwidth equal to the data rate. This can be reduced by passing the modulating signal through a low pass filter prior to applying it to the carrier. The requirements for the filter are that it should have a sharp cut-off, narrow bandwidth and its impulse response should show no overshoot. The ideal filter is known as a Gaussian filter which has a Gaussian shaped response to an impulse and no ringing. In this way the basic MSK signal is converted to GMSK modulation.

In GMSK, the side lobe levels of the spectrum are reduced by passing the NRZ data waveform through a premodulation Gaussian pulse-shaping filter.



The GMSK premodulation filter has an impulse response given by

$$h_G(t) = \frac{\sqrt{\pi}}{\alpha}\exp\left(-\frac{\pi^2}{\alpha^2}t^2\right)$$

and the transfer function given by

$$H_G(f) = \exp(-\alpha^2 f^2)$$

The parameter α is related to B [the 3-dB baseband bandwidth of $H_G(f)$] by

$$\alpha = \frac{\sqrt{\ln 2}}{\sqrt{2}B}$$

The bit error probability is a function of BT, since the pulse shaping impacts ISI. The bit error probability for GMSK is given by

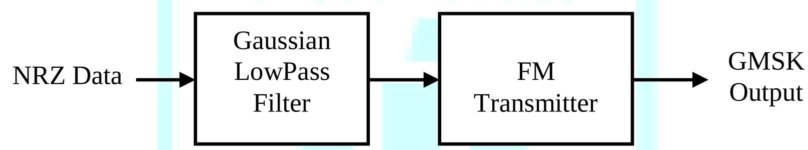
$$P_e = Q\left(\sqrt{\frac{2\gamma E_b}{N_0}}\right)$$

where

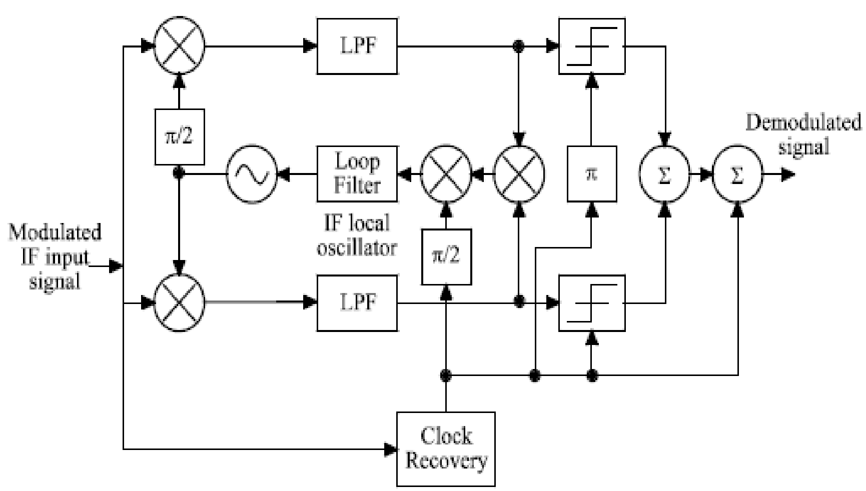
$$\gamma = \begin{cases} 0.68 \text{ for } GMSK \text{ with BT = 0.25} \\ 0.85 \text{ for simple MSK (BT} = \infty) \end{cases}$$

GMSK Transmitter and Receiver

There are two main ways in which GMSK modulation can be generated. The most obvious way is to filter the modulating signal using a Gaussian filter and then apply this to a frequency modulator where the modulation index is set to 0.5. This method is very simple and straightforward but it has the drawback that the modulation index must exactly equal 0.5. In practice this analogue method is not suitable because component tolerances drift and cannot be set exactly.



A second method is more widely used. Here what is known as a quadrature modulator is used. The term quadrature means that the phase of a signal is in quadrature or 90 degrees to another one. The quadrature modulator uses one signal that is said to be in-phase and another that is in quadrature to this. In view of the in-phase and quadrature elements this type of modulator is often said to be an I-Q modulator. Using this type of modulator the modulation index can be maintained at exactly 0.5 without the need for any settings or adjustments. This makes it much easier to use, and capable of providing the required level of performance without the need for adjustments. For demodulation the technique can be used in reverse.



Properties of GMSK

Desirable properties of MSK are

- Constant envelope
- GMSK can be coherently detected, or noncoherently detected.
- GMSK has excellent power efficiency (due to the constant envelope)
- GMSK has excellent spectral efficiency
- The sidelobe levels of the spectrum are reduced by passing the NRZ data waveform through a Gaussian pulse-shaping filter.

Advantages of GMSK modulation

There are several advantages to the use of GMSK modulation for a radio communications system. One is obviously the improved spectral efficiency when compared to other phase shift keyed modes.

A further advantage of GMSK is that it can be amplified by a non-linear amplifier and remain undistorted. This is because there are no elements of the signal that are carried as amplitude variations.

This advantage is of particular importance when using small portable transmitters, such as those required by cellular technology. Non-linear amplifiers are more efficient in terms of the DC power input from the power rails that they convert into a radio frequency signal. This means that the power consumption for a given output is much less, and this results in lower levels of battery consumption; a very important factor for cell phones.

A further advantage of GMSK modulation again arises from the fact that none of the information is carried as amplitude variations. This means that is immune to amplitude variations and therefore more resilient to noise, than some other forms of modulation, because most noise is mainly amplitude based.

Disadvantage

The premodulation Gaussian filtering introduces ISI in the transmitted signal, but the degradation is not severe if the 3 dB-bandwidth-bit duration product (BT) of the filter is greater than 0.5.

METHODS FOR COMPUTATION OF ERROR PROBABILITY

The **bit error rate** is the number of bit errors per unit time. The **bit error ratio** is the number of errors divided by the number of transmitted bits.

Modulation formats can all be classified easily

1. BPSK signals are antipodal signals.
2. BFSK, and Binary Pulse Position Modulation (BPPM), are orthogonal signals.
3. QPSK, π/4-DQPSK, and OQPSK are bi-orthogonal signals.

Error Probability for Coherent Receivers – General Case

Coherent receivers compensate for phase rotation of the channel by means of **carrier recovery**. Furthermore, the channel gain α is assumed to be known.

The probability that symbol s_j is mistaken for symbol s_k is given by

$$\Pr_{pair}(\mathbf{s}_j, \mathbf{s}_k) = Q\left(\sqrt{\frac{d_{jk}^2}{2N_0}}\right)$$

where d_{jk} is Euclidean distance between jth and kth signal points in constellation.

Error Probability for Coherent Receivers – Binary Orthogonal Signals

Binary Frequency Shift Keying (FSK) and binary Pulse Position Modulation (PPM) can be viewed as binary orthogonal signals.

The pairwise error probability is:

$$\Pr_{pair}(\mathbf{s}_j, \mathbf{s}_k) = Q\left(\sqrt{\frac{E_S}{N_0}}\right)$$

Defining the Signal-to-Noise Ratio (SNR) for one symbol as $\gamma_S = \frac{E_S}{N_0}$, we get,

$$\Pr_{pair}(\mathbf{s}_j, \mathbf{s}_k) = Q(\sqrt{\gamma_S})$$

Note that since we are considering binary signaling $\gamma_S = \gamma_B$.

Error Probability for Coherent Receivers – Antipodal Signaling

For antipodal signals, the pairwise error probability is:

$$\Pr_{pair}(\mathbf{s}_j, \mathbf{s}_k) = Q(\sqrt{2\gamma_S})$$

For binary signals with equal-probability transmit symbols, pairwise error probability is equal to symbol error probability, which in turn is equal to bit error probability. This is the case, e.g., for BPSK, as well as for MSK with ideal coherent detection.

Error Probability for Differential Detection

Carrier recovery can be a challenging problem which makes coherent detection difficult for many situations. Differential detection is an attractive alternative to coherent detection.
For binary orthogonal signals, the BER for differential detection is

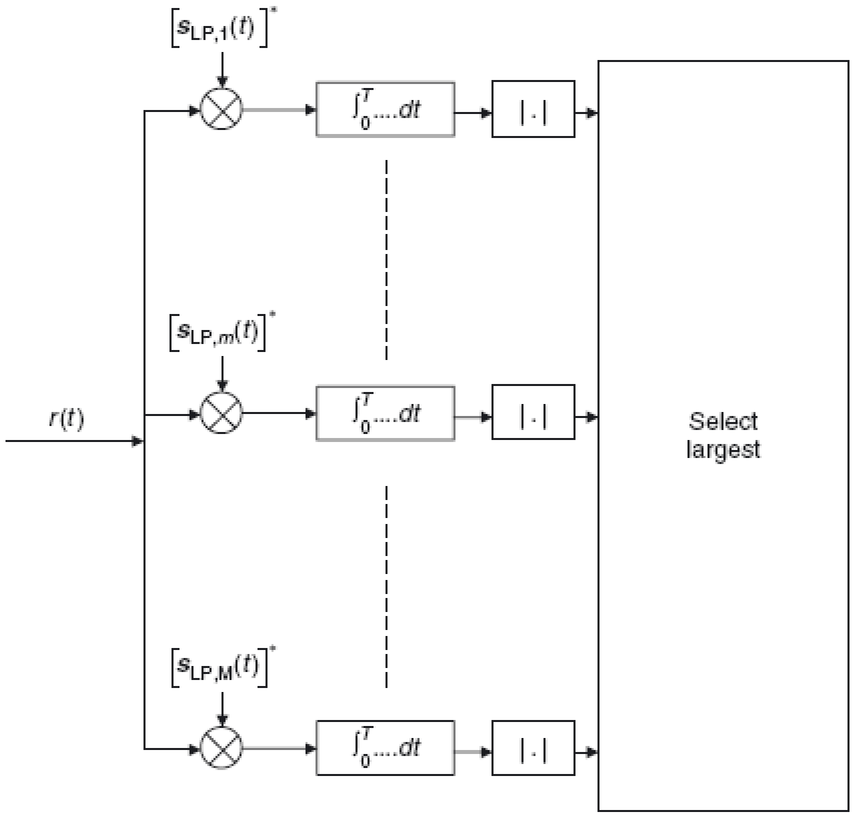
$$BER = \frac{1}{2}\exp(-\gamma_B)$$

Error Probability for Noncoherent Detection

When the carrier phase is completely unknown, and differential detection is not an option, then noncoherent detection can be used.
For equal-energy signals, the detector tries to maximize the metric:

$$\left|\mathbf{r}_{LP}\mathbf{s}_{LP,m}^*\right|$$

So that the optimum receiver has a structure shown below.



An actual realization, where $r(t)$ is a bandpass signal, each branch of figure splits the signal into two subbranches, in which it obtains and processes the I- and Q-branches of the signal separately. The outputs of the absolute value operation of the I- and Q-branches are then added up before the select largest operation

ERROR PROBABILITY IN FLAT-FADING CHANNELS

Slow, flat fading channels change much slower than the applied modulation. So it can be assumed that the attenuation and phase shift of the signal is constant over at least one symbol interval.

Therefore, the received signal r(t) may be expressed as

$$r(t) = \alpha(t) e^{-j\theta(t)} s(t) + n(t)$$

where

- $\alpha(t)$ is the gain of the channel
- $\theta(t)$ is the phase shift of the channel
- $n(t)$ is additive Gaussian noise.

For a mathematical computation of the BER in a channel, we have to proceed in three steps:

1. Determine the BER for any arbitrary SNR.
2. Determine the probability that a certain SNR occurs in the channel.
3. Average the BER over the distribution of SNRs.

The mean BER in a slow, flat fading channel can be evaluated as

$$\overline{BER} = \int pdf_{\gamma_B}(\gamma_B) BER(\gamma_B)\, d\gamma_B$$

where,

- $pdf_{\gamma_B}(\gamma_B)$ is the pdf of SNR (γ_B), and $\gamma_B = \alpha^2 E_b/N_0$
- $BER(\gamma_B)$ is the BER for an arbitrary modulation at a spenfic value of SNR
- the random variable α is used to represent amplitude values of the fading channel

The pdf of the SNR is

$$pdf_{\gamma_B}(\gamma_B) = \frac{1}{\overline{\gamma}_B} \exp\left(-\frac{\gamma_B}{\overline{\gamma}_B}\right)$$

The mean BER for coherent detection of binary antipodal signals is

$$\overline{BER} = \frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma}_B}{1+\overline{\gamma}_B}}\right] \cong \frac{1}{4\overline{\gamma}_B}$$

The mean BER for coherent detection of binary orthogonal signals is

$$\overline{BER} = \frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma}_B}{2+\overline{\gamma}_B}}\right] \cong \frac{1}{2\overline{\gamma}_B}$$

For differential binary antipodal signals:

$$\overline{BER} = \frac{1}{4\overline{\gamma}_B} = \frac{1}{2(1+\overline{\gamma}_B)} \cong \frac{1}{2\overline{\gamma}_B}$$

For differential binary orthogonal signals

$$\overline{BER} = \frac{1}{2+\overline{\gamma}_B} \cong \frac{1}{\overline{\gamma}_B}$$

Orthogonal Frequency Division Multiplexing

Orthogonal Frequency Division Multiplexing or OFDM is a modulation format that is being used for many of the latest wireless and telecommunications standards.

OFDM has been adopted in the Wi-Fi arena where the standards like 802.11a, 802.11n, 802.11ac and more. It has also been chosen for the cellular telecommunications standard LTE / LTE-A, and in addition to this it has been adopted by other standards such as WiMAX and many more.

Orthogonal frequency division multiplexing has also been adopted for a number of broadcast standards from DAB Digital Radio to the Digital Video Broadcast standards, DVB. It has also been adopted for other broadcast systems as well including Digital Radio Mondiale used for the long medium and short wave bands.

Although OFDM, orthogonal frequency division multiplexing is more complicated than earlier forms of signal format, it provides some distinct advantages in terms of data transmission, especially where **high data rates are needed along with relatively wide bandwidths.**

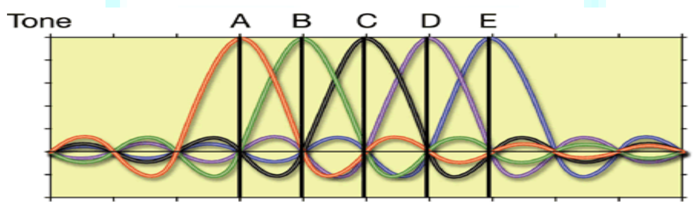
PRINCIPLE OF OFDM

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi-carrier modulation scheme that extends the concept of single subcarrier modulation by using multiple subcarriers within the same single channel. Rather than transmit a high-rate stream of data with a single subcarrier, OFDM makes use of a large number of closely spaced orthogonal subcarriers that are transmitted in parallel. Each subcarrier is modulated with a conventional digital modulation scheme (such as QPSK, 16QAM, etc.) at low symbol rate. However, the combination of many subcarriers enables data rates similar to conventional single-carrier modulation schemes within equivalent bandwidths.

OFDM is based on the well-known technique of Frequency Division Multiplexing (FDM). In FDM different streams of information are mapped onto separate parallel frequency channels. Each FDM channel is separated from the others by a frequency guard band to reduce interference between adjacent channels.

The OFDM scheme differs from traditional FDM in the following interrelated ways:

1. Multiple carriers (called subcarriers) carry the information stream,
2. The subcarriers are orthogonal to each other, and
3. A guard interval is added to each symbol to minimize the channel delay spread and intersymbol interference.



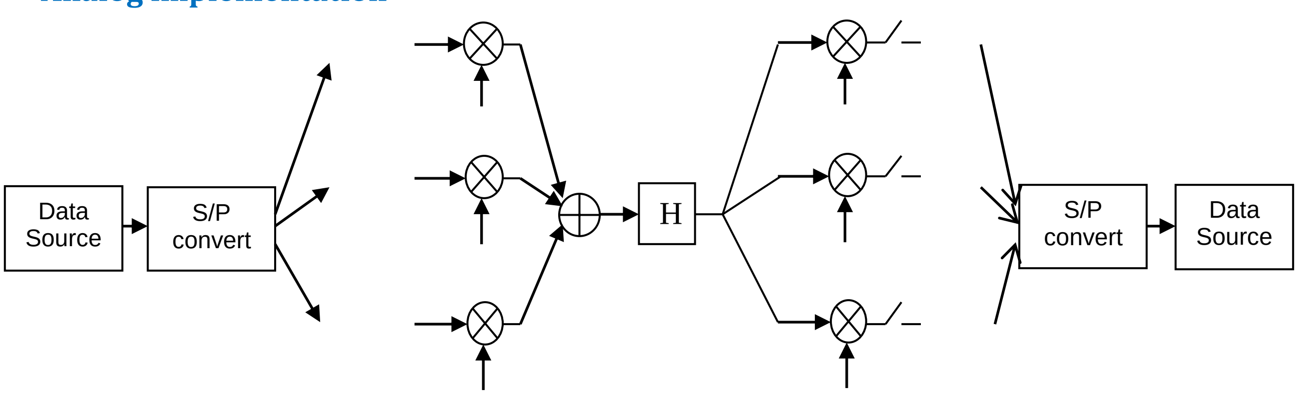
OFDM splits a high-rate data stream into N parallel streams, which are then transmitted by modulating N distinct carriers (called **subcarriers** or **tones**). Symbol duration on each subcarrier thus becomes larger by a factor of N.

In order for the receiver to separate signals carried by different subcarriers, they have to be orthogonal.

The subcarriers can be at the frequencies $f_n = nW/N$, where n is an integer, and W the total available bandwidth.

IMPLEMENTATION OF TRANSCEIVERS

Analog implementation



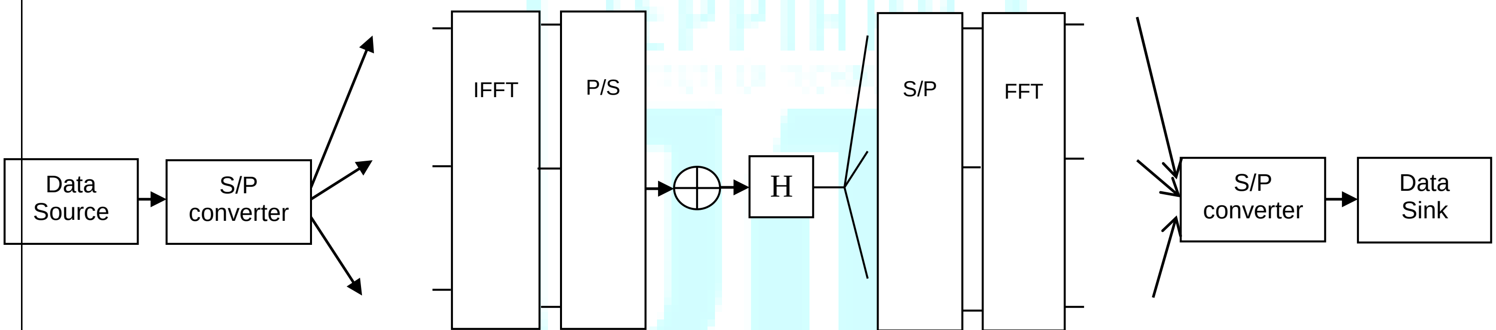
The original data stream is split into N parallel data streams, each of which has a lower data rate. There are number of local oscillators (LOs) available, each of which oscillates at a frequency $f_n = nW/N$, where $n = 0, 1, \ldots, N-1$. Each of the parallel data streams then modulates one of the carriers.

Analog implementation is not suited for actual implementation – the hardware effort of multiple local oscillators is too high.

Digital implementation

The digital implementation of OFDM is shown below.

It first divides the transmit data into blocks of N symbols. Each block of data is subjected to an **Inverse Fast Fourier Transformation** (IFFT), and then transmitted. This approach is much easier to implement with integrated circuits.



Let us first consider the analog interpretation. Let the complex transmit symbol at time instant i on the nth carrier be $c_{n,i}$. The transmit signal is then:

$$s(t) = \sum_{i=-\infty}^{\infty} s_i(t) = \sum_{i=-\infty}^{\infty} \sum_{n=0}^{N-1} c_{n,i} g_n(t - iT_S)$$

where the basis pulse $g_n(t)$ is

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T_S}} \exp\left(j2\pi n \frac{t}{T_S} \right) & \text{for } 0 < t < T_S \\ 0 & \text{otherwise} \end{cases}$$

Consider the signal only for $i = 0$, and sample it at instances $t_K = kT_S/N$:

$$s_k = s(t_k) = \frac{1}{\sqrt{T_S}} \sum_{n=0}^{N-1} c_{n,0} \exp\left(j2\pi n \frac{k}{N} \right)$$

This is Inverse Discrete Fourier Transform (IDFT) of the transmit symbols.

Therefore, the transmitter can be realized by performing an **Inverse Discrete Fourier Transform** on the block of transmit symbols. Input to this IFFT is made up of N samples and therefore the output from the IFFT also consists of N values. These N values now have to be transmitted, one after the other, as temporal samples. This is done by a P/S (Parallel to Serial) conversion directly after the IFFT.

At the receiver, the process is reversed: The received signal is first sampled and converted into a block of N using an S/P (Serial to Parallel) converter. An FFT is then performed on this block. The result is an estimate $\tilde{c}_n$ of the original data c_n.

The digital implementation of the OFDM transceiver is much simpler and cheaper.

OFDM ADVANTAGES & DISADVANTAGES

OFDM advantages

OFDM has been used in many high data rate wireless systems because of the many advantages it provides.

- ***Immunity to selective fading:*** One of the main advantages of OFDM is that is more resistant to frequency selective fading than single carrier systems because it divides the overall channel into multiple narrowband signals that are affected individually as flat fading sub-channels.
- ***Resilience to interference:*** Interference appearing on a channel may be bandwidth limited and in this way will not affect all the sub-channels. This means that not all the data is lost.
- ***Spectrum efficiency:*** Using close-spaced overlapping sub-carriers, a significant OFDM advantage is that it makes efficient use of the available spectrum.
- ***Resilient to ISI:*** Another advantage of OFDM is that it is very resilient to inter-symbol and inter-frame interference. This results from the low data rate on each of the sub-channels.
- ***Resilient to narrow-band effects:*** Using adequate channel coding and interleaving it is possible to recover symbols lost due to the frequency selectivity of the channel and narrow band interference. Not all the data is lost.
- ***Simpler channel equalisation:*** One of the issues with CDMA systems was the complexity of the channel equalisation which had to be applied across the whole channel. An advantage of OFDM is that using multiple sub-channels, the channel equalization becomes much simpler.

OFDM disadvantages

Whilst OFDM has been widely used, there are still a few disadvantages to its use which need to be addressed when considering its use.

- ***High peak to average power ratio:*** An OFDM signal has a noise like amplitude variation and has a relatively high large dynamic range, or peak to average power ratio. This impacts the RF amplifier efficiency as the amplifiers need to be linear and accommodate the large amplitude variations and these factors mean the amplifier cannot operate with a high efficiency level.

- ***Sensitive to carrier offset and drift:*** Another disadvantage of OFDM is that is sensitive to carrier frequency offset and drift. Single carrier systems are less sensitive.

FREQUENCY-SELECTIVE CHANNELS

Cyclic Prefix

Delay dispersion leads to

1. appreciable errors even when delay spread < symbol duration.
2. loss of orthogonality between the subcarriers, and thus leads to **Inter Carrier Interference.**

Both these negative effects can be eliminated by a special type of guard interval, called the **cyclic prefix**.

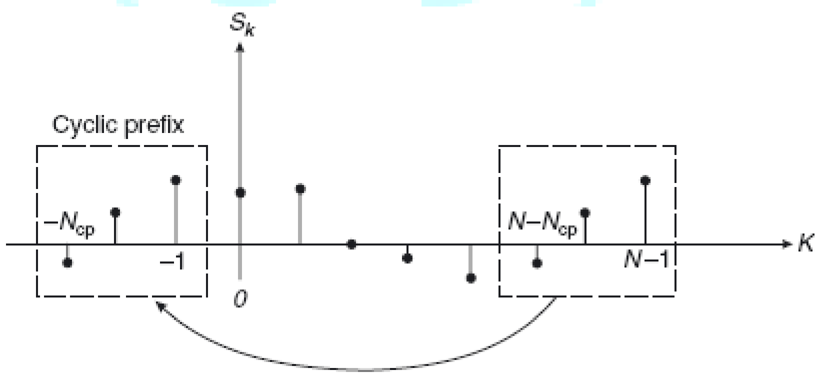
The Cyclic Prefix (CP) helps to eliminate residual delay dispersion.

Define a new base function for transmission:

$$g_n(t) = \exp\left(j2\pi n \frac{W}{N} t \right) \quad \text{for } -T_{cp} < t < \hat{T}_S$$

where W/N is the carrier spacing,

The symbol duration T_S is now $T_S = \hat{T}_S + T_{cp}$. So for duration $0 < t < \hat{T}_S$ the normal OFDM symbol is transmitted. During time $-T_{cp} < t < 0$, a copy of the last part of the symbol is transmitted.



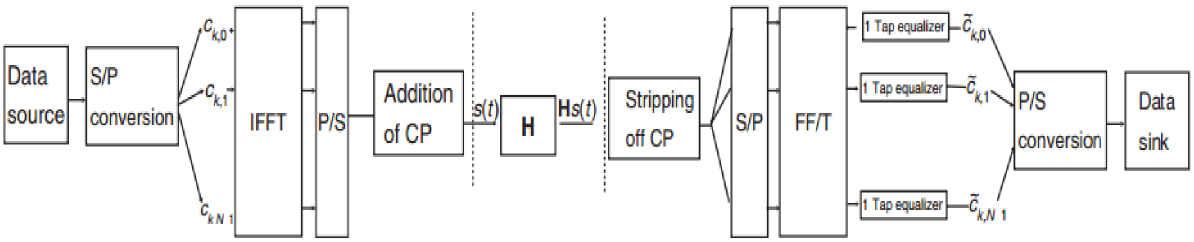
The total signal $s(t)$ transmitted during time $-Tcp < t < 0$ is a copy of $s(t)$ during the last part, $\hat{T}_S - T_{cp} < t < \hat{T}_S$ .This prepended part of the signal is called the **cyclic prefix**.

When transmitting any data stream over a delay-dispersive channel, the arriving signal is the linear convolution of the transmitted signal with the channel impulse response. The cyclic prefix converts this *linear* convolution into a *cyclical* convolution.

During the time $-T_{cp} < t < -T_{cp}+\tau_{max}$, where τ_{max} is the maximum excess delay of the channel, the received signal suffers from *real* ISI, the last part of the preceding symbol interfere with the desired symbol. This *regular* ISI is eliminated by discarding the received signal during this time interval.

During the remainder of the symbol, we have *cyclical* ISI. Especially, it is the last part of the current symbol that interferes with the first part of the current symbol.

The block diagram of an OFDM system, including the cyclic prefix, is given in Figure



The original data stream is S/P converted. Each block of N data symbols is subjected to an IFFT, and then the last NT_{cp}/T_S samples are prepended. The resulting signal is modulated onto a (single) carrier and transmitted over a channel, which distorts the signal and adds noise.

At the receiver, the signal is partitioned into blocks. For each block, the cyclic prefix is stripped off, and the remainder is subjected to an FFT. The resulting are equalized by means of one-tap equalization on each carrier.

Peak-to-Average Power Ratio (PAPR)

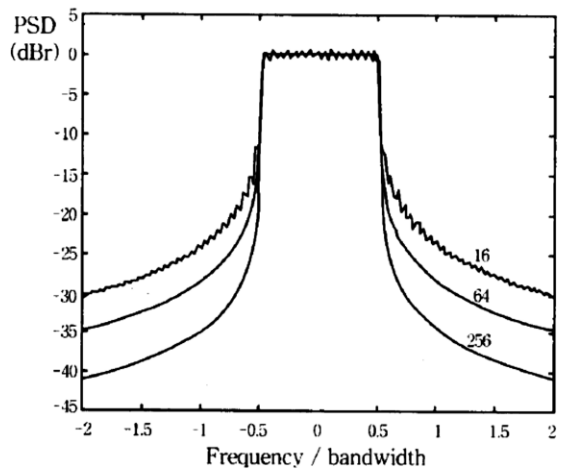
Peak-to-average power ratio (PAPR) is proportional to the number of subcarriers used for OFDM systems. An OFDM system with large number of subcarriers will thus have a very large PAPR when the subcarriers add up coherently. Large PAPR of a system makes the implementation of digital-to-analog converter (DAC) and analog-to-digital converter (ADC) extremely difficult. The design of RF amplifier also becomes increasingly difficult as the PAPR increases.

The clipping and windowing technique reduces PAPR by non-linear distortion of the OFDM signal. It thus introduces self-interference as the maximum amplitude level is limited to a fixed level. It also increases the out-of-band radiation, but this is the simplest method to reduce the PAPR. To reduce the error rate, additional forward error correcting codes can be used in conjunction with the clipping and windowing method.
Another technique called linear peak cancelation can also be used to reduce the PAPR. In this method, time-shifted and time-scaled reference function is subtracted from the signal, such that each subtracted reference function reduces the peak power of at least one signal sample. By selecting an appropriate reference function with approximately the same bandwidth as the transmitted function, it can be assured that the peak power reduction does not cause out-of-band interference. One example of a suitable reference function is a raised cosine window

Windowing

An OFDM signal consists of a number of unfiltered sub-carriers.

Therefore, the out-of-band spectrum decreases rather slowly, with the speed depending on the number of sub-carriers, as shown below.



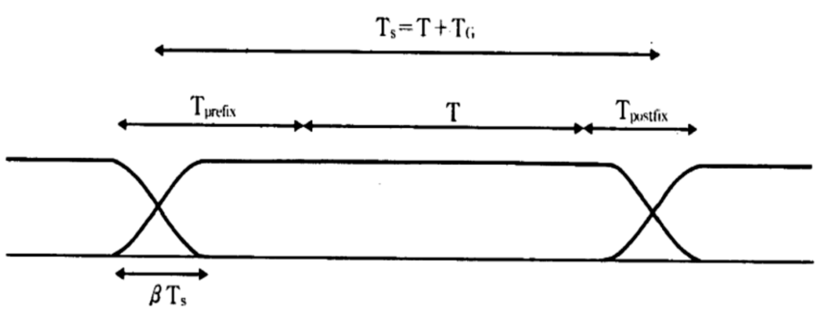
PSD without windowing for 16, 64, 256 sub-carriers

To make the spectrum go down more rapidly, windowing can be applied to individual OFDM symbols.

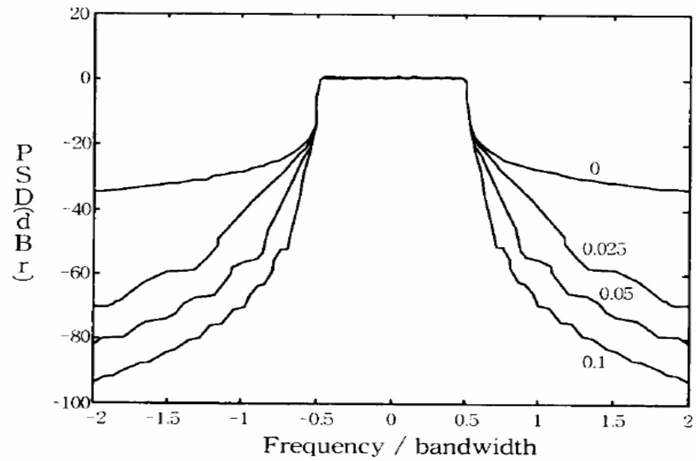
Mostly used is the raised cosine window, which is defined as

$$w(t)=\begin{cases} 0.5+0.5\cos\left(\pi+t\pi/(\beta T_s)\right), & 0\le t\le \beta T_s \\ 1.0, & \beta T_s \le t\le T_s \\ 0.5+0.5\cos\left((t-T_s)\pi/(\beta T_s)\right), & T_s\le t\le (1+\beta)T_s \end{cases}$$

where β is the roll-off factor



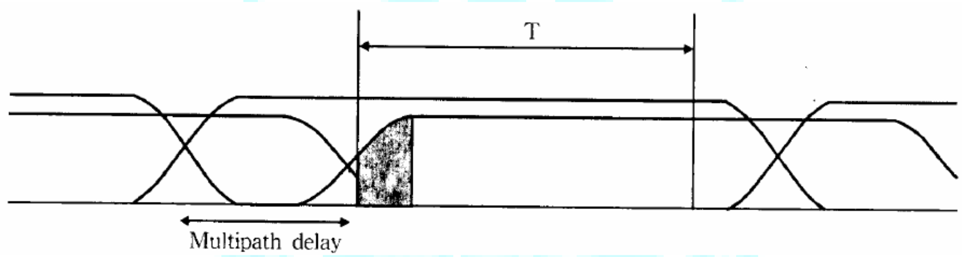
OFDM cyclic extension and windowing



Spectra for raised cosine windowing with roll-off factor 0, 0.025, 0.05, and 0.1 (64 sub-carriers)

Larger roll-off factors improve the spectrum further, at the cost, however, of a decreased delay spread tolerance.

Instead of windowing, it is possible to use virtual carriers by nulling the subcarriers around the edge of the spectrum.



OFDM symbol windows for a two-ray multi-path channel, showing ICI and ISI