**DEPARTMENT**
**OF**
**ELECTRICAL AND ELECTRONICS ENGINEERING**

# LECTURE NOTES

## EE8451- ELECTROMAGENTIC FIELDS

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## UNIT III MAGNETOSTATICS

**Introduction**

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated.

**Laws governing magneto static fields**

The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later. Historically, the link between the electric and magnetic field was established Oersted in 1820. Ampere and others extended the investigation of magnetic effect of electricity . There are two major laws governing the magnetostatic fields are:

Biot-Savart Law

Ampere's Law

Usually, the magnetic field intensity is represented by the vector $\vec{H}$. It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with a dot or cross sign depending on whether the field (or current) is out of or into the page as shown in Fig. 3.1.

$\vec{H}$ (or l ) out of the page $\vec{H}$ (or l ) into the page

**Fig. 3.1: Representation of magnetic field (or current) Biot- Savart Law**

This law relates the magnetic field intensity *dH* produced at a point due to a differential current element $Id\vec{l}$ as shown in Fig. 3.2.

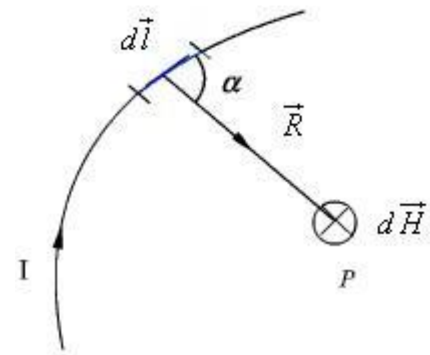



**Fig. 3.2: Magnetic field intensity due to a current element**

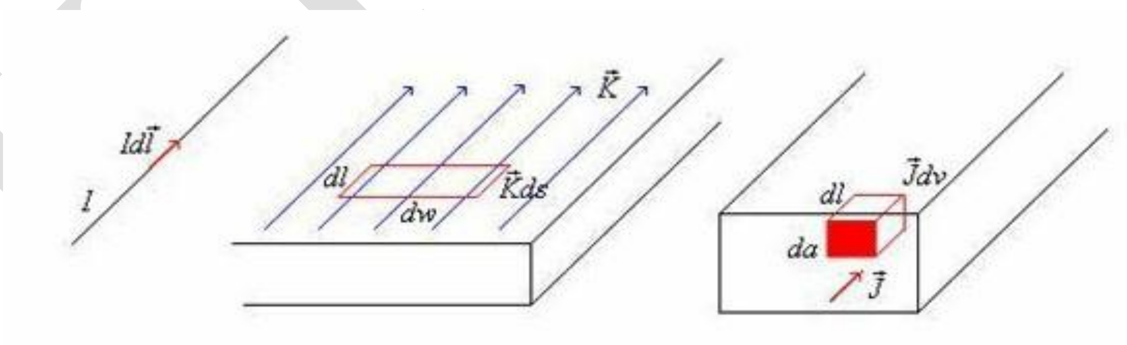
The magnetic field intensity $d\vec{H}$ at P can be written as,

$$d\vec{H} = \frac{Id\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3} \quad \text{..........................(3.1a)}$$

$$dH = \frac{IdlSin\alpha}{4\pi R^2} \quad \text{...........................................(3.1b)}$$

where $R = \left|\vec{R}\right|$ is the distance of the current element from the point P.

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 3.3.




Line Current Surface Current Volume Current

**Fig. 3.3: Different types of current distributions**

By denoting the surface current density as K (in amp/m) and volume current density as J (in amp/m$^2$) we can write:

$$I\vec{dl} = \vec{K}ds = \vec{J}dv \quad ......................................(3.2)$$

( It may be noted that $I = Kdw = Jda$ )

Employing Biot-Savart Law, we can now express the magnetic field intensity H. In terms of these current distributions.

$$\vec{H} = \int_l \frac{I\vec{dl} \times \vec{R}}{4\pi R^3}$$ ... **for line current..........................** (3.3a)

$$\vec{H} = \int_s \frac{K\vec{ds} \times \vec{R}}{4\pi R^3}$$ **for surface current....................**(3.3b)

$$\vec{H} = \int_v \frac{\vec{J}dv \times \vec{R}}{4\pi R^3}$$ **for volume current....................** (3.3c)

To illustrate the application of Biot - Savart's Law, we consider the following example.

**Example 3.1:** We consider a finite length of a conductor carrying a current $\vec{r}$ placed along z- axis as shown in the Fig 3.4. We determine the magnetic field at point P due to this current carrying conductor.

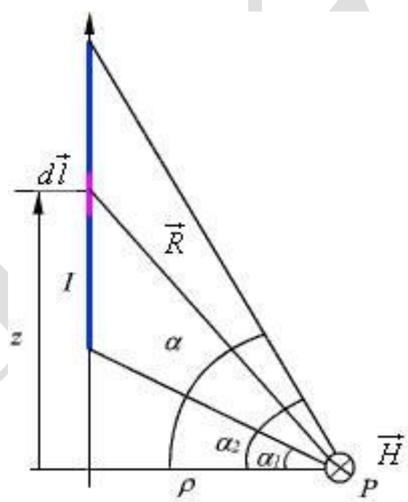



**Fig. 3.4: Field at a point P due to a finite length current carrying conductor**

With reference to Fig. 4.4, we find that

$$\vec{dl} = dz\,\hat{a}_z \quad and \quad \vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$$

..................................................(3.4)

Applying Biot -Savart's law for the current element $\vec{r}\,\vec{dl}$

we can write,

$$\overrightarrow{dH} = \frac{I d\vec{l} \times \vec{R}}{4\pi R^3} = \frac{\rho dz \hat{a}_\phi}{4\pi[\rho^2 + z^2]^{3/2}} \quad \text{................(3.5)}$$

Substituting $\frac{z}{\rho} = \tan\alpha$ we canwrite,

$$\vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{I}{4\pi} \frac{\rho^2 \sec^2 \alpha d\alpha}{\rho^3 \sec^3 \alpha} \hat{a}_\phi = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi \quad \text{.........................(3.6)}$$

We find that, for an infinitely long conductor carrying a current I , $\alpha_2 = 90^0$ and $\alpha_1 = -90^0$

Therefore, $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$ ............................................................................................................ (3.7)

**Ampere's Circuital Law:**

Ampere's circuital law states that the line integral of the magnetic field $\vec{H}$ (circulation of *H* ) around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H}.d\vec{l} = I_{enc} \quad \text{.....................................(3.8)}$$

The total current I enc can be written as,

$$I_{enc} = \int_S \vec{J}.d\vec{s} \quad \text{......................................(3.9)}$$

By applying Stoke's theorem, we can write

$$\oint \vec{H}.d\vec{l} = \int_S \nabla \times \vec{H}.d\vec{s}$$

$$\therefore \int_S \nabla \times \vec{H}.d\vec{s} = \int_S \vec{J}.d\vec{s}$$

$$\therefore \nabla \times \vec{H} = \vec{J} \quad \text{......................................(3.10)}$$

which is the Ampere's law in the point form.

**Estimation of Magnetic field intensity forstraight and circular conductors:**

We illustrate the application of Ampere's Law with some examples.

We compute magnetic field due to an infinitely long thin current carrying conductor as shown in Fig. 4.5. Using Ampere's Law, we consider the close path to be a circle of radius as $\rho$ shown in the Fig. 4.5.

If we consider a small current element $I d\vec{l}(= I dz \hat{a}_z)$ , $d\vec{H}$ is perpendicular to the plane containing both $d\vec{l}$ and $\vec{R}(=\rho\hat{a}_\rho)$ . Therefore only component of $\vec{H}$ that will be present is $H_\phi$,i.e., $\vec{H} = H_\phi \hat{a}_\phi$ .

By applying Ampere's law we can write,

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \int_0^{2\pi} H_\phi \rho d\phi = H_\phi \rho 2\pi = I \quad \text{.................(4.11)}$$

Therefore, $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$ which is same as equation (3.7)

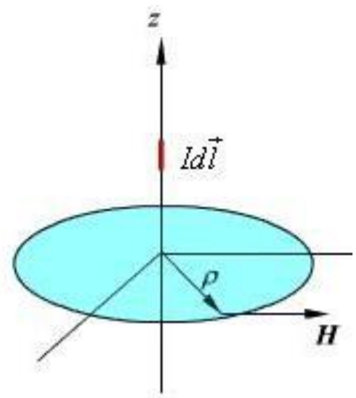



**Fig. 3.5: Magnetic field due to an infinite thin current carrying conductor**

We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current I and outer conductor carrying current - I as shown in figure 3.6. We compute the magnetic field as a function of $\rho$ as follows:

In the region $0 \le \rho \le R_1$

$$I_{enc} = I\frac{\rho^2}{R_1^2} \quad \text{.....................................(3.12)}$$

$$H_\phi = \frac{I_{enc}}{2\pi\rho} = \frac{I\rho}{2\pi a^2} \quad \text{............................(3.13)}$$

In the region $R_1 \le \rho \le R_2$

$I_{enc} = I$

$$H_\phi = \frac{I}{2\pi\rho} \quad \text{.....................................(3.14)}$$

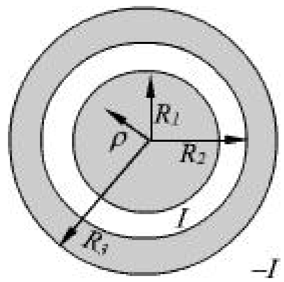



Fig. 3.6: Coaxial conductor carrying equal and opposite currents

In the region $R_2 \le \rho \le R_3$

$$I_{enc} = I - I\frac{\rho^2 - R_2^2}{R_3^2 - R_2^2} \quad \text{.....................................(3.15)}$$

$$H_\phi = \frac{I}{2\pi\rho}\frac{R_3^2 - \rho^2}{R_3^2 - R_2^2} \quad \text{.....................................(3.16)}$$

In the region $\rho > R_3$ $\quad I_{enc} = 0 \qquad H_\phi = 0$ .....................................(3.17)

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**Magnetic Flux and Density:**

In simple matter, the magnetic flux density $\vec{B}$ related to the magnetic field intensity $\vec{H}$ as $\vec{B} = \mu\vec{H}$ where $\mu$ called the permeability. In particular when we consider the free space $\vec{B} = \mu_0\vec{H}$ where $\mu_0 = 4\pi\times10^{-7}$ H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m $^2$ .

The magnetic flux density through a surface is given by:

$$\psi = \int_S \vec{B}.d\vec{s} \quad \text{Wb} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.18)$$

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i. e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig. 3.7 (a). This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.

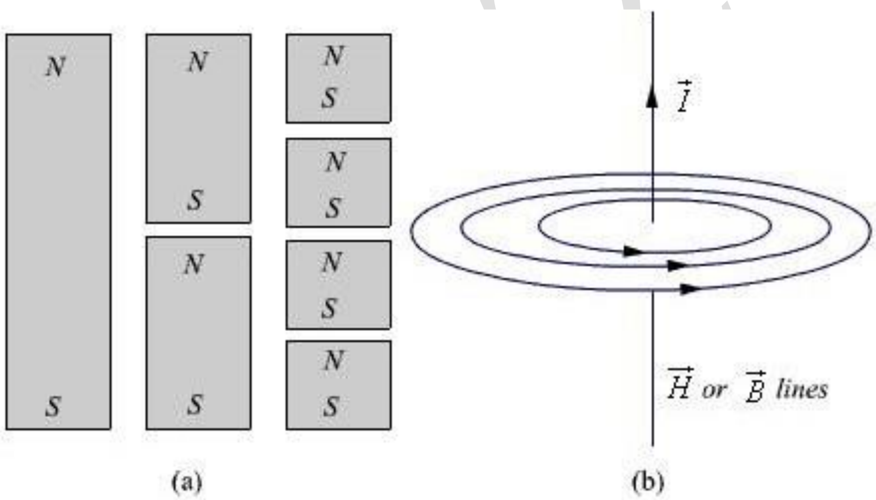


**Fig. 3.7: (a) Subdivision of a magnet (b) Magnetic field/ flux lines of a straight current carrying conductor**

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig. 3.7 (b), we find that these lines are closed lines, that is, if we consider a closed surface, the number of flux lines that would leave the surface would be same as the number of flux lines that would enter the surface.

From our discussions above, it is evident that for magnetic field,

$$\oint_S \vec{B}.d\vec{s} = 0 \quad \text{.....................................(3.19)}$$

which is the Gauss's law for the magnetic field. By applying divergence theorem, we can write:

$$\oint_S \vec{B}.d\vec{s} = \int_V \nabla.\vec{B}dv = 0$$

Hence, ........................................................................................................ $\nabla.\vec{B} = 0$ (3.20)

which is the Gauss's law for the magnetic field in point form.

**Magnetic Scalar and Vector Potentials:**

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a **scalar magnetic potential** and write:

$\vec{H} = -\nabla V_m$ ..................................(3.21)

From Ampere's law , we know that

$$\nabla \times \vec{H} = \vec{J} \quad \text{.....................................(3.22)}$$

Therefore, ........................................................................................ $\nabla \times (-\nabla V_m) = \vec{J}$ (3.23)

But using vector identity, $\nabla \times (\nabla V) = 0$ we find that $\vec{H} = -\nabla V_m$ is valid only where $\vec{J} = 0$. Thus the scalar magnetic potential is defined only in the region where $\vec{J} = 0$. Moreover, *Vm* in general is not a single valued function of position.

This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig 3.8.

In the region $a < \rho < b$ , $\vec{J} = 0$ and $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$

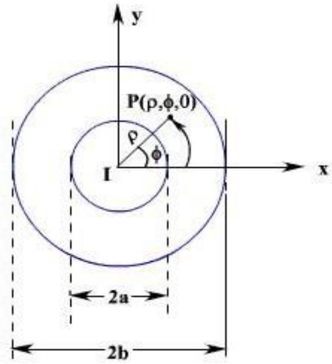


**Fig. 3.8: Cross Section of a Coaxial Line**

If *Vm* is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi}$$
$$= \frac{I}{2\pi\rho}$$

$$\therefore V_m = -\frac{I}{2\pi}\phi + c$$

If we set *Vm* = 0 at $\phi = 0$ then c=0 and $V_m = -\frac{I}{2\pi}\phi$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi}\phi_0$$

We observe that as we make a complete lap around the current carrying conductor , we reach $\phi_0$ again but *Vm* this time becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi)$$

We observe that value of *Vm* keeps changing as we complete additional laps to pass through the same point. We introduced *Vm* analogous to electostatic potential *V*. But for static electric fields, $\nabla \times \vec{E} = 0$ and $\oint \vec{E}\cdot d\vec{l} = 0$, whereas for steady magnetic field $\nabla \times \vec{H} = 0$ wherever $\vec{J} = 0$ but $\oint \vec{H}\cdot d\vec{l} = I$ even if $\vec{J} = 0$ along the path of integration.

We now introduce the **vector magnetic potential** which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.